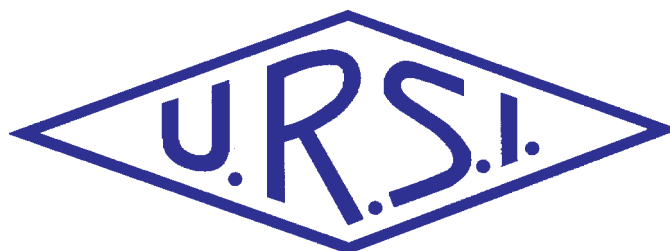


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A Perspective on Distributed Radio Systems with Cooperative Coding



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Abstract

In distributed radio systems, e.g., wireless sensor or ad-hoc networks, the system's performance may be significantly enhanced by means of cooperation among the nodes. In this paper, we present a perspective on *cooperative coding* in distributed radio systems. In particular, we consider a simple reference scenario where two source nodes need to transmit two correlated information sequences to a common access point (AP). This is a scenario of interest for wireless sensor networks, where the sensor may observe correlated phenomena. Four possible cooperative-coding situations are described: (i) a non-cooperative system (NCS), (ii) cooperative source coding (CSC), (iii) cooperative source-channel coding (CSCC), and (iv) joint source-channel coding (JSCC). While in the first scheme source correlation is not used at all, the other schemes differ according to the way source correlation is used: from cooperative source coding systems, where source correlation is used only at the sources, to joint source-channel coding systems, where source correlation is used only at the access point. Indeed, joint source-channel coding systems are attractive in scenarios (such as wireless scenarios) where communications between the sources might be problematic. As an illustrative example, we will present a practical joint source-channel coding system in the presence of block-faded channels, using low-density parity-check (LDPC) coding at the sources and a proper iterative decoder at the access point.

1. Introduction and Motivation

In this paper, we focus on distributed communication systems where two or more nodes need to transmit to a common remote destination. This model applies to many scenarios, such as cellular networks, wireless local-area networks with one access point (AP), ad-hoc wireless networks, wireless sensor networks, etc. In these scenarios, collaboration between the nodes might bring significant advantages in terms of *collaborative diversity* [1]. In a cooperative system, each user is assigned one or more partners. The partners overhear each other, process the received signals, and retransmit proper messages to the destination in order to provide extra information to the access point with respect to the signal sent by a single source. Even in the presence of noisy inter-partner channels, the virtual transmitting-antenna array formed by cooperating nodes provides additional diversity, and may improve the system's performance in terms of error rate and throughput.

In the literature, many schemes have been proposed to exploit collaborative diversity. These schemes differ, especially for the relaying technique used, i.e., on the basis of the information that is re-transmitted by cooperating nodes to guarantee the highest ratio between diversity degree and resource consumption. The simplest schemes are those where the nodes re-transmit all the received information in an orthogonal way (typically, with time-division multiplexing): the codes used are not very efficient, but the highest diversity is guaranteed [2-5]. In other schemes, only a concise version of the information received by a cooperating node is transmitted, e.g., a parity bit [6]. Finally, there are

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schemes where the nodes simultaneously access the shared radio medium – typically modeled as a Gaussian multiple-access channel (GMAC) – with Alamouti-like space-time coding [7, 8]. In this case, the first direct transmission corresponds to the first row of the Alamouti code matrix (which, nevertheless, corresponds to transmissions at different moments, since the nodes cannot transmit and receive at the same time), while the simultaneous transmissions are associated with the second row. In the latter case, the nodes have only to transmit, so that the transmissions can be simultaneous. A scheme of this type allows a much higher efficiency than the previous schemes, since the multiple-access interference is completely eliminated, owing to the orthogonality of the Alamouti matrix; obviously, perfect synchronization between the nodes is required. In classical cooperation scenarios, the idea is therefore that of making the nodes cooperate among themselves to implement a distributed channel-coding scheme, where different nodes retransmit the same information, in some sense.

However, in many application scenarios the information that resides in different nodes is *intrinsically correlated*. In other words, even without implementing any cooperation among the nodes, the same or, more generally, “similar” information is transmitted by the nodes. A significant application example where this situation typically appears is given by *wireless sensor networks* [9]. In this case, the design of efficient transmission of correlated signals, observed at different nodes, to one or more collectors is one of the main design challenges. In the presence of one collector node, this system model is often referred to as a reach-back channel [10-12]. In its simplest form, this problem can be summarized as follows. Two independent nodes have to transmit correlated sensed data to a collector node by using the minimum possible energy, i.e., by exploiting in some way implicit correlation among the data. In the case of orthogonal additive white Gaussian noise (AWGN) channels, the separation between source and channel coding is known to be optimal [13, 14]. This means that the theoretical limit can be achieved by first compressing each source up to the Slepian-Wolf (SW) limit, and then utilizing two independent capacity-achieving channel codes (one per source) [15]. In this case, no cooperation among the source nodes is required.

However, if the transmissions are not carried out through separate additive white Gaussian noise channels, then the Slepian-Wolf approach is no longer optimal. Alternative schemes, which encompass the presence of cooperating nodes, can bring significant advantages. In [16], a Gaussian multiple-access channel scheme was considered. First, the nodes exchange information through time-division-based transmission acts and taking into account the correlation, i.e., they transmit much less with respect to the entire information by relying on a distributed-source coding-based approach. Once each source node has the entire information (relative to both source nodes), it then compresses and retransmits it to the destination node (the

access point), thus achieving a beamforming gain with respect to a scenario with no cooperation. This approach basically requires that the information is first compressed, thus exploiting the correlation between the sources, and then duplicated in order to obtain a coding and diversity, or beamforming, gain.

An alternative solution to exploit the correlation in this scenario is based on joint-source channel-coding (JSCC) schemes, where no cooperation among nodes is required, and the correlated sources are not source encoded but only channel encoded. The absence of direct cooperation between the source nodes is attractive in scenarios where the communication links between the source nodes may be noisy. If one compares a joint-source channel-coding system with a system based on source/channel-coding separation with the same information rate, the channel codes used in a joint-source channel-coding scheme must be less powerful (i.e., they have higher rates). This weakness can be compensated for by exploiting the correlation between the sources at the decoder, which jointly recovers the information signals by both source nodes, so that the final performance can approach the theoretical limits. For this reason, this approach is also referred to as joint channel decoding (JCD). This approach has attracted the attention of several researchers in the recent past, also because of its implementation simplicity [17-21]. Note that in the joint-source channel-coding approach, the sources are encoded independently of each other (i.e., for a given source, neither the realization from the other source nor the correlation model are available at the encoder), and transmitted through the channel. Correlation between the sources must be instead assumed to be known at the (common) receiver.

In this context, the scheme proposed in [16], with “intrinsic” cooperation, is of particular interest. The basic idea is (i) to let the two sources transmit simultaneously two correlated code words, and (ii) to let the decoder solve the bit erasures (which appear when the transmitted bits are different). In [16], it was shown that this scheme may achieve a 3 dB potential beamforming gain, but it requires perfect synchronization and perfect channel-state information (CSI) at the transmitters. A scheme of this type does not therefore seem feasible in the presence of a channel affected by multipath fading, where guaranteeing perfect channel-state information at the transmitters requires a supplementary signaling load, which cannot be sustained, in many cases. Hence, in the presence of multipath fading, the problem of designing suitable (and reliable) non-cooperative joint source-channel coding schemes so that an “intrinsic diversity gain” can be achieved at the decoder by exploiting the side information (i.e., the a priori correlation between the information sequences) is still an open issue.

In this work, we present a simple perspective on cooperative-coding strategies for distributed radio systems. For ease of derivation, we will introduce a simple reference scenario (two source nodes and a common access point), which will allow us to analytically evaluate the performance

of various cooperative-transmission schemes in the presence or absence of explicit cooperation. In all cases, we consider the presence of block-faded channels and power control – under the constraint of maximum transmitted power. In particular, we first analyze various schemes where the source correlation is exploited at the source nodes and/or at the access point. Considering a joint-source channel-coding scheme in the case of orthogonal multiple access, we will introduce the concept of *correlation-induced* diversity gain, to be compared with *cooperation-induced* diversity gain. Our results show that in many cases, the presence of correlation between sources limits the necessity for explicit cooperation between them. Finally, we will present a “practical” example of a low-density parity-check (LDPC) coded joint-source channel-coding scheme.

2. Two-Source-Node Scenario

We consider the distributed radio system shown in Figure 1. The correlated information sequences at the source nodes are indicated by \mathbf{x} and \mathbf{y} , respectively. This scenario may correspond to a scheme where two sensor nodes, denoted as SN_1 and SN_2 , detect the two correlated signals \mathbf{x} and \mathbf{y} . These signals are assumed to be independent and identically distributed (i.i.d.) correlated binary random variables, with

$$Pr(x_i = 0) = Pr(x_i = 1) = 0.5,$$

$$Pr(y_i = 0) = Pr(y_i = 1) = 0.5,$$

for $i = 0, \dots, k-1$ and $\rho \triangleq Pr(x_i = y_i) > 0.5$.

The information signals, which are assumed to be detectable without error (i.e., ideal sensor nodes), must be delivered to the access point. To this aim, each sensor node establishes a direct link toward the access point and an indirect link toward the other sensor node, in order to exploit cooperative transmission. We assume that transmissions from the nodes to the access point and between

nodes occur over orthogonal channels (e.g., by using time-division multiple access). Moreover, we assume that the communication links are all affected by multiplicative fading and additive white Gaussian noise. Referring to the equivalent low-pass signal representation, and considering digital transmission, we denote as s_x (s_y) the complex transmitted sequence corresponding to the information signal \mathbf{x} (\mathbf{y}), with α being the complex link-gain term, which encompasses both path loss and fading, and with \mathbf{n} being the complex additive white Gaussian noise sequence.

Each source node transmits N symbols every k information symbols, so that $r \triangleq k/N$ corresponds to the effective transmission rate of each source node (this might encompass the presence of both source and channel coding, as will be described later). In more general terms, the problem at hand consists of transmitting $2k$ information symbols through $2N$ channel uses. Each source node can communicate to the access point and to the other source node. We assume that all transmissions are performed in a highly scattered environment, without line of sight (NLOS). Hence, the channel complex gains, denoted as α_x (direct link from SN_1 to the access point), α_y (direct link from SN_2 to the access point), α_{xy} (direct link from SN_1 to SN_2), and α_{yx} (direct link from SN_2 to SN_1) can be modeled as zero-mean Gaussian random variables (Rayleigh fading). The fading coefficients in the direct links, i.e., α_x and α_y , are supposed to be independent (this is reasonable if the two sensor nodes are more than a wavelength away), while the fading coefficients in the indirect (inter-sensor) links are supposed to be equal, i.e., $\alpha_{xy} = \alpha_{yx}$ (this is reasonable if only one carrier frequency is used for both transmissions and the source nodes are quasi-static). Slow fading and path loss are assumed to have the same statistical distribution for both direct links.

Note that in the presence of channel fluctuations, an optimal transmission scheme should encompass a joint power-control and link-adaptation mechanism to adapt both the transmission rates and the transmitted powers to the actual channel conditions, so that the ultimate capacity may be achieved. However, combining power control with link adaptation is a difficult task. Specifically, without knowing the transmission power beforehand, the SNR cannot be predicted, as it would be needed to choose the

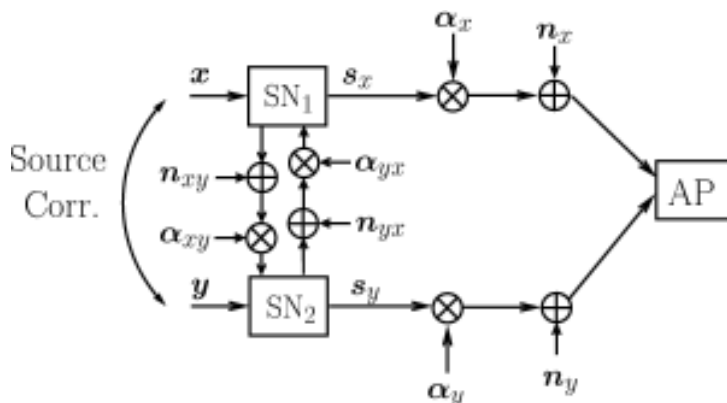


Figure 1. A distributed radio system where two source nodes need to transmit their information to a common access point and can communicate between themselves

Table 1. The main quantities used in the derivations

Quantity	Definition
$P_x = E\left(s_y ^2\right) \left[P_y = \mathbb{E}\left(s_y ^2\right) \right]$	power transmitted by SN ₁ [SN ₂] in the direct link
$P_{xy} = \mathbb{E}\left(s_{xy} ^2\right) \left[P_{yx} = \mathbb{E}\left(s_{yx} ^2\right) \right]$	power transmitted by SN ₁ [SN ₂] in the indirect link
R_x [R_y]	number of bits per detected sample transmitted by SN ₁ [SN ₂] in the direct link
R_{xy} [R_{yx}]	number of bits per detected sample transmitted by SN ₁ [SN ₂] in the indirect link
$G_x = \alpha_x ^2$ [$G_y = \alpha_y ^2$]	power gain term for the direct link of SN ₁ [SN ₂]
$G_{xy} = \alpha_{xy} ^2$ [$G_{yx} = \alpha_{yx} ^2$]	power gain term for the indirect link of SN ₁ [SN ₂]
$\sigma_d^2 = \mathbb{E}\left(n_x ^2\right) = \mathbb{E}\left(n_y ^2\right)$	AWGN variance in the direct links
$\sigma_i^2 = \mathbb{E}\left(n_{xy} ^2\right) = \mathbb{E}\left(n_{yx} ^2\right)$	AWGN variance in the indirect link
$\gamma_x = \frac{G_x}{\sigma_d^2}$ [$\gamma_y = \frac{G_y}{\sigma_d^2}$]	SNR (normalized to the transmitted power) for the direct link of SN ₁ [SN ₂]
$\gamma_{xy} = \frac{G_{xy}}{\sigma_i^2}$ [$\gamma_{yx} = \frac{G_{yx}}{\sigma_i^2}$]	SNR (normalized to the transmitted power) for the indirect link of SN ₁ [SN ₂]
$\Gamma_x = \Gamma_y = \mathbb{E}\left(G_x/\sigma_d^2\right)$	average SNR (normalized to the transmitted power) for the direct links
$\Gamma_{xy} = \Gamma_{yx} = \mathbb{E}\left(G_{xy}/\sigma_i^2\right)$	average SNR (normalized to the transmitted power) for the indirect links
$f_\gamma^{(y)}(u) = f_\gamma^{(x)}(u) = \frac{1}{\Gamma_x} e^{-\frac{u}{\Gamma_x}} U(u)$	common Rayleigh pdf of γ_x and γ_y
$f_\gamma^{(yx)}(u) = f_\gamma^{(xy)}(u) = \frac{1}{\Gamma_{xy}} e^{-\frac{u}{\Gamma_{xy}}} U(u)$	common Rayleigh pdf of γ_{xy} and γ_{yx}
$P_{max}^{(d)}$ [$P_{max}^{(i)}$]	maximum transmitted power in the direct [indirect] links, normalized to the inverse of the average SNR (i.e., $P_{max}^{(d)} = 1$ [$P_{max}^{(i)} = 1$] means that the maximum power is equal to $1/\Gamma_x$ [$1/\Gamma_{xy}$])

appropriate modulation/coding level. In turn, without knowing the modulation level, the transmitted power cannot be adjusted accordingly. Hence, in the following we assume that the transmission rates are fixed (i.e., no link adaptation), while the transmitted power can be adapted by means of an ideal closed power-control mechanism, which adjusts the transmitted power to a level sufficiently high to achieve desirable performance. In particular, the feedback power-adjustment messages sent by the access point are received without errors, and each sensor can set its transmitted power within a predefined range. The assumption of ideal power control in the presence of faded links may not be practical in the case of fast fading. In fact, the use of feedback power control requires the presence of very reliable return links

(from the access point to the sources), to convey the channel-state information back to the transmitter, and a limited number of feedback commands. The impact of practical feedback-control strategies is beyond the scope of this paper (see [22] for more details). Instead, we take into account limitations in the transmitted power range by introducing $P_{max}^{(d)}$ and $[P_{max}^{(i)}]$ as the maximum transmitted power in the direct [indirect] links, normalized to the inverse of the average SNR (i.e., $P_{max}^{(d)} = 1$ [$P_{max}^{(i)} = 1$] means that the maximum power is equal to $1/\Gamma_x$ [$1/\Gamma_{xy}$]).

For the sake of notational simplicity, in Table 1 we summarize the main quantities used in the remainder of this paper.

3. A Perspective on Possible Approaches

We now outline possible approaches to performing cooperative coding in distributed radio systems with correlated sources. These approaches, illustrated in Figure 2, can be summarized as follows:

- Non-cooperative system (NCS): the source correlation is *not* exploited at the sources and the access point;
- Cooperative source coding (CSC): the source correlation is exploited at the *sources*;
- Cooperative source-channel coding (CSCC): the source correlation is exploited at *both the source and the access point*;
- Joint source-channel coding (JSCC): the source correlation is exploited at the *access point*.

For each approach, the probability of incorrect delivery of the information signals from both sources, denoted as probability of error, will be derived. At the end, a comparative performance analysis is proposed.

3.1 First Approach: Non-Cooperative System

In this case, the two sensor nodes perform independent channel coding and the access point performs independent channel decoding, i.e., the source is not exploited at all. In this case, $R_x = R_y = 1$, i.e., the number of bits that must be transmitted by each sensor node every N channel uses is k . There is no multiple-access interference, i.e., the direct links are orthogonal (for example, time-division multiple access is considered). According to the Shannon channel-capacity formula, the maximum *common* transmit rate has to satisfy the following expressions:

$$r = \frac{1}{2} \log_2 \left(1 + \frac{P_x G_x}{\sigma_d^2} \right),$$

$$r = \frac{1}{2} \log_2 \left(1 + \frac{P_y G_y}{\sigma_d^2} \right),$$

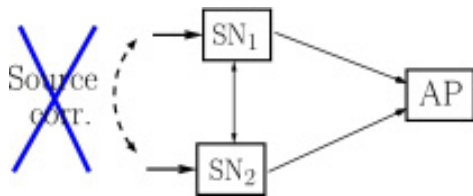


Figure 2a. A possible approach to exploiting the source correlation in a two-source-node distributed radio system: a non-cooperative system (NCS)

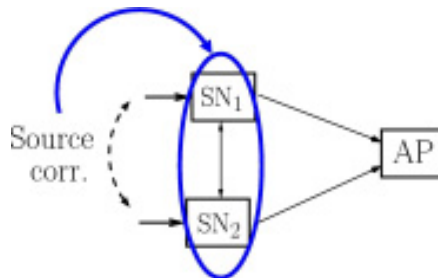


Figure 2b. A possible approach to exploiting the source correlation in a two-source-node distributed radio system: cooperative source coding (CSC).

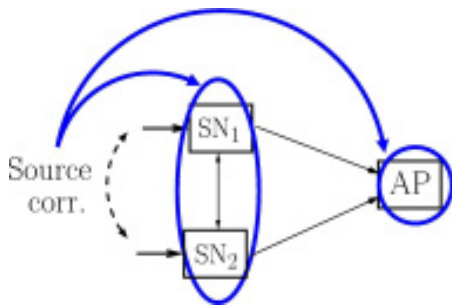


Figure 2c. A possible approach to exploiting the source correlation in a two-source-node distributed radio system: cooperative source-channel coding (CSCC).

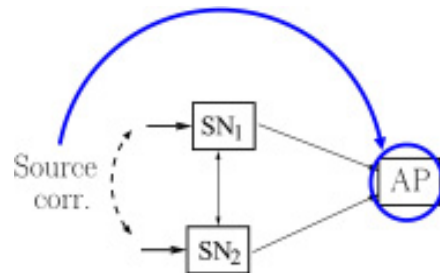


Figure 2d. A possible approach to exploiting the source correlation in a two-source-node distributed radio system: joint source-channel coding (JSCC).

from which the following expressions for the transmitted powers can be obtained:

$$P_x = (2^{2r} - 1) \frac{\sigma_d^2}{G_x} = (2^{2r} - 1) \frac{1}{\gamma_x},$$

$$P_y = (2^{2r} - 1) \frac{\sigma_d^2}{G_y} = (2^{2r} - 1) \frac{1}{\gamma_y}.$$

Given the distribution of the SNRs, γ_x and γ_y , and for the sake of notational simplicity defining the quantity $W_1 \triangleq 2^{2r} - 1$, the average transmitted powers become

$$\mathbb{E}(P_x) = \int_{\frac{W_1 \Gamma_x}{P_{max}^{(d)}}}^{\infty} \frac{W_1}{u} \frac{1}{\Gamma_x} e^{-\frac{u}{\Gamma_x}} du = \frac{W_1}{\Gamma_x} \int_{\frac{W_1}{P_{max}^{(d)}}}^{\infty} \frac{1}{u} e^{-u} du,$$

$$\mathbb{E}(P_y) = \mathbb{E}(P_x).$$

The total average energy (normalized to the symbol time) is

$$E_t = 2N \mathbb{E}(P_x).$$

Eventually, it is possible to evaluate the probability that a correct delivery of both information signals is not possible because of limited power resources. This probability, denoted as Pr_E , is the complement of the probability that both transmissions can be successfully carried out, i.e.,

$$Pr_E = 1 - \left(\int_{\frac{W_1}{P_{max}^{(d)}}}^{\infty} \frac{1}{\Gamma_x} e^{-\frac{u}{\Gamma_x}} du \right)^2 = 1 - e^{-\frac{2W_1}{P_{max}^{(d)}}}.$$

3.2 Second Approach: Cooperative Source Coding

In this case, the two sensor nodes perform cooperative source coding and independent channel coding, whereas the access point performs independent channel decoding. It is worth to note that owing to the Slepian-Wolf theorem, ideal cooperative source coding can be achieved without any transmission between the two source nodes, i.e., without using the indirect links. Note that following the assumption of fixed transmitting rates, the transmitters

are forced to select a compression rate that corresponds to one corner of the Slepian-Wolf region, leaving to closed-loop power control full responsibility for combating fading fluctuations. In this scenario, assuming that SN₁ transmits the information signal at full rate $R_x = 1$ (i.e., no compression is performed), SN₂ may transmit the “ $y | x$ ” information at the following rate:

$$R_y = R_{y|x} = H(y | x)$$

$$= -\log_2(\rho)\rho - \log_2(1-\rho)(1-\rho).$$

As expected, the numbers of information bits that must be delivered by the two sensor nodes in this case are different. More precisely, $R_x > R_y$. Hence, the solution of assigning the same number of samples, N , to each source node is not the optimal solution. Denote by η ($0 \leq \eta \leq 1$) the percentage of time assigned to SN₁: the use of time-division multiple access (i.e., orthogonal channels) is assumed in this case, as well. In this case, taking into account that the access point performs independent decoding per each direct link, one obtains

$$\frac{r}{2\eta} = \frac{1}{2} \log_2(1 + P_x \gamma_x),$$

$$\frac{rR_{y|x}}{2(1-\eta)} = \frac{1}{2} \log_2(1 + P_y \gamma_y),$$

from which the following expressions for the transmitted powers can be derived:

$$P_x = \left(2^{\frac{r}{2\eta}} - 1 \right) \frac{1}{\gamma_x},$$

$$P_y = \left(2^{\frac{rR_{y|x}}{2(1-\eta)}} - 1 \right) \frac{1}{\gamma_y}.$$

Following the same considerations as the previous section, and for the sake of notational simplicity defining $W_2 \triangleq 2^{r/\eta} - 1$ and $W_3 \triangleq 2^{rR_{y|x}/(1-\eta)} - 1$, it is straightforward to derive the following expressions for the average transmitted powers:

$$\mathbb{E}(P_x) = \frac{W_2}{\Gamma_x} \int_{\frac{W_2}{P_{max}^{(d)}}}^{\infty} \frac{1}{u} e^{-u} du,$$

$$\mathbb{E}(P_y) = \frac{W_3}{\Gamma_x} \int_{\frac{W_3}{P_{max}^{(d)}}}^{\infty} \frac{1}{u} e^{-u} du.$$

The total average energy (normalized to the symbol time) can be finally written as

$$E_t = 2N \left[\eta \mathbb{E}(P_x) + (1-\eta) \mathbb{E}(P_y) \right].$$

The error probability, Pr_E , in this case is

$$Pr_E = 1 - \exp \left\{ -\frac{W_2 + W_3}{P_{max}^{(d)}} \right\}.$$

3.3 Third Approach: Cooperative Source-Channel Coding

In this case, the two sensor nodes cooperate to exploit the potential benefit brought about by the use of space-time coding for multiple-input multiple-output (MIMO) systems. According to this approach, originally proposed in [16], two source nodes need to transmit the same information bits¹. Hence, as a first step, SN_1 transmits its information signal, x , to SN_2 . SN_2 then needs to transmit its information signal, y , to SN_1 . Of course, by invoking the Slepian-Wolf theorem, these transmissions can be performed at the reduced rates $R_{x|y}$ and $R_{y|x}$, respectively. Note that due to the symmetry of the problem, $R_{x|y} = R_{y|x}$.

Assume now that the two indirect inter-source transmission steps take a percentage of time δ (with respect to the overall time spent to transmit to the access point), i.e., $\delta/2$ for the transmission from SN_1 to SN_2 , and $\delta/2$ for the other transmission. In this case, one can write:

$$\frac{rR_{x|y}}{\delta} = \frac{1}{2} \log_2 (1 + P_{xy} \gamma_{xy}),$$

$$\frac{rR_{y|x}}{\delta} = \frac{1}{2} \log_2 (1 + P_{yx} \gamma_{yx}),$$

which yields

$$P_{xy} = \left(2^{\frac{2rR_{x|y}}{\delta}} - 1 \right) \frac{1}{\gamma_{xy}},$$

$$P_{yx} = \left(2^{\frac{2rR_{y|x}}{\delta}} - 1 \right) \frac{1}{\gamma_{yx}}.$$

Following the same considerations as the previous sections, for the sake of notational simplicity defining $W_4 \triangleq 2^{2rR_{x|y}/\delta} - 1$ and $\vartheta = \Gamma_{xy}/\Gamma_x = \Gamma_{yx}/\Gamma_y$, and noting that $2^{rR_{x|y}/\delta} - 1 = 2^{rR_{y|x}/\delta} - 1$, the average transmitted powers can be straightforwardly expressed as

$$\mathbb{E}(P_{xy}) = \frac{W_4}{\Gamma_{xy}} \int_{\frac{\vartheta W_4}{P_{max}^{(d)}}}^{\infty} \frac{1}{u} e^{-u} du,$$

$$\mathbb{E}(P_{yx}) = \mathbb{E}(P_{xy}).$$

Following the same derivation as in the previous section, the probability of correct transmission in the indirect links, denoted as $Pr_C^{(i)}$, can be expressed as

$$Pr_C^{(i)} = \exp \left\{ -\frac{2\vartheta W_4}{P_{max}^{(d)}} \right\}.$$

After this first transmission step, the two nodes can send the same information signal, (x, y) at the rate $1 + R_{x|y}$ by using a MIMO transmission scheme, thus achieving the ideal MIMO capacity; obviously, the two source nodes have to be synchronous. We remark that the multiple-access interference can be completely eliminated due to the structure of the Alamouti matrix. The MIMO capacity formula for the 2×1 transmission scheme depends on the correlation properties of the channel vector (α_x, α_y) . On the basis of the independence assumption introduced in Section 2, it can immediately be concluded that the capacity is the same as the two-degree diversity case. Hence, observing that the remaining transmissions in the direct links can be performed simultaneously by the two nodes in the remaining $1 - \delta$ fraction of time, and that the two nodes use the same transmission power, $P_t/2$ (water-filling transmission schemes are not considered), one finally obtains

$$\frac{r(R_x + R_{x|y})}{2(1-\delta)} = \frac{1}{2} \log_2 \left[1 + \frac{P_t (\gamma_x + \gamma_y)}{2} \right],$$

from which it follows that

$$P_t = \left[2^{\frac{r(R_x + R_{x|y})}{2(1-\delta)}} - 1 \right] \frac{2}{\gamma_x + \gamma_y}.$$

In order to evaluate the average transmission power, it is now necessary to derive the probability density function (pdf) of the sum of the SNRs, $\gamma_x + \gamma_y$. By the independence assumption, it can be observed that $\gamma_x + \gamma_y$ has a central chi-square distribution with four degrees of freedom and a mean of $\Gamma_x + \Gamma_y$. Hence, its probability density function is

$$f_{\gamma_x + \gamma_y}(u) = \frac{u}{(\Gamma_x + \Gamma_y)^2} e^{-\frac{u}{\Gamma_x + \Gamma_y}} = \frac{u}{4\Gamma_x^2} e^{-\frac{u}{2\Gamma_x}}.$$

Observing that the constraint on the maximum power in this case is $P_t \leq 2P_{max}^{(d)}/\Gamma_x$, and defining the quantity $W_5 \triangleq 2^{r(R_x + R_{x|y})[2(1-\delta)]} - 1$, the average transmitted power, P_t , can be expressed in closed form as follows:

$$\begin{aligned} \mathbb{E}(P_t) &= \int_{\frac{\Gamma_x W_5}{P_{max}^{(d)}}}^{\infty} \frac{2W_5}{\Gamma_x W_5} \frac{u}{4\Gamma_x^2} e^{-\frac{u}{2\Gamma_x}} du \\ &= \frac{W_5}{\Gamma_x} \int_{\frac{W_5}{2P_{max}^{(d)}}}^{\infty} e^{-u} du \\ &= \frac{W_5}{\Gamma_x} e^{-\frac{W_5}{2P_{max}^{(d)}}}. \end{aligned}$$

The probability of correct transmission in the direct link can then be computed as follows:

$$\begin{aligned} Pr_C^{(d)} &= Pr \left\{ P_t \leq 2 \frac{P_{max}^{(d)}}{\Gamma_x} \right\} \\ &= \int_{\frac{\Gamma_x W_5}{P_{max}^{(d)}}}^{\infty} \frac{u}{4\Gamma_x^2} e^{-\frac{u}{2\Gamma_x}} du \\ &= e^{-\frac{W_5}{2P_{max}^{(d)}}} \left(1 + \frac{W_5}{2P_{max}^{(d)}} \right). \end{aligned}$$

Taking into account all (direct and indirect) transmissions, the total average energy is

$$E_t = 2N \left[\delta \mathbb{E}(P_{xy}) + (1-\delta) \mathbb{E}(P_t) \right],$$

and the probability of error can be expressed as

$$Pr_E = 1 - Pr_C^{(i)} Pr_C^{(d)}.$$

3.4 Fourth Approach: Joint Source-Channel Coding

In this case, the two transmitters perform independent channel coding at the same (compressing) rate, $R=1$. In other words, the correlation is not exploited at the transmitters, while the access point performs joint channel decoding, i.e., the correlation is exploited at the receiver. In the case of orthogonal direct channels, the achievable (common for both source nodes) channel rate, r , has to satisfy the following inequalities [21]:

$$rR_{y|x} \leq \frac{1}{2} \log_2 \left(1 + \frac{P_x G_x}{\sigma_d^2} \right),$$

$$rR_{x|y} \leq \frac{1}{2} \log_2 \left(1 + \frac{P_y G_y}{\sigma_d^2} \right),$$

$$r(1 + R_{x|y}) - \frac{1}{2} \log_2 \left(1 + \frac{P_x G_x}{\sigma_d^2} \right) - \frac{1}{2} \log_2 \left(1 + \frac{P_y G_y}{\sigma_d^2} \right) \leq 0.$$

By solving the above system of inequalities for given values of r and ρ (equivalently, for given values of r , $R_{x|y}$, and $R_{y|x}$), it is possible to determine the feasibility region of the system in the bi-dimensional normalized SNRs bi-dimensional space (γ_x, γ_y) . In Figure 3, an illustrative example is given by the region with the curve denoted as “theoretical limit” as the lower bound, in a scenario with $r=0.5$ and $\rho=0.9$. More details of this figure will be presented later. At this point, the probability of error for a fixed maximum transmitted power can be computed through Monte Carlo simulations, according to the following steps:

1. Fix the maximum transmitted energy, E_t , (or, equivalently, the maximum transmitted power for a given value of r) from each node.
2. Generate two fading realizations, G_x and G_y .
3. Under the assumption of ideal power control, determine the possible couples of transmitted powers required for the current fading realizations, in order for the actual SNRs to belong to the feasibility region. Since there are

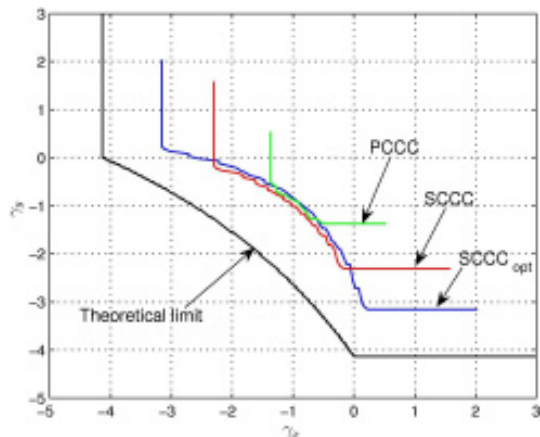


Figure 3. The feasible region (the lower bound of which is given by the curve denoted as “theoretical limit”) in the bi-dimensional SNR space (γ_x, γ_y) . The achievable regions, under the use of various channel-coding strategies, derived in [23] are also shown.

infinite couples of transmitted powers that allow the SNR couples to belong to the feasibility region, pick the couple of transmitted powers with minimum value of the maximum of the two.

4. The previously selected couple of transmitted powers will either satisfy the constraint on the maximum transmit power, or not. Under the assumption of ideal channel coding, if the constraint on the maximum transmitted power is satisfied, then there is no error; otherwise, there is an error.
5. By repeating the above steps a sufficiently large number of times, the probability of error can be numerically evaluated.

In the above derivation, we have assumed ideal channel coding. We now comment on this idealistic assumption. By relaxing the assumption of ideal channel coding, it is possible to evaluate the performance of specific channel

codes with proper receiver structures. As mentioned in Section 1, there is intense research activity on this topic [17-21]. While in most of this work the presence of memory-less direct links was considered, in [24], various coded and uncoded schemes were analyzed in the presence of block-faded channels. In Section 4, an illustrative low-density parity-check-coded scheme, proposed in [24], will be recalled, and its performance evaluated. At this point, an interesting question is, “Does there exist an “optimal” channel code?” In order to answer this question, in [23] an innovative two-dimensional extrinsic information transfer (EXIT) chart-inspired optimization approach was proposed. This approach allows the determination of the feasibility region, in the channel SNRs bi-dimensional space, associated with the particular coding scheme under use. These results suggest that by properly designing serially concatenated convolutional codes (SCCCs), or parallel concatenated convolutional codes (PCCCs), the theoretical performance limits can be approached. In [23], preliminary results were shown in the presence of low-density parity-check coding, but their accurate optimization is one of our current research activities. In Figure 3, the feasibility regions in the case of additive white Gaussian noise communication channels² are shown, considering various turbo codes. The interested reader can find more details in [23]. Approaching the theoretical limits very closely remains an open problem.

3.5 Comparative Performance Evaluation

In Figure 4, the average transmitted energy, E_t , is shown as a function of ρ , considering the four possible types of schemes outlined before in this section, under the condition that $Pr_E = 0.01$. In the cooperative source-channel coding scheme, three values of θ , namely $\theta = -3$, $\theta = 0$, and $\theta = 3$ dB, are considered. In all cases, the average values $\Gamma_x = \Gamma_y = 10$ dB. Note that for $\theta > 0$ dB, i.e., when the SNR in the indirect link is higher than in the

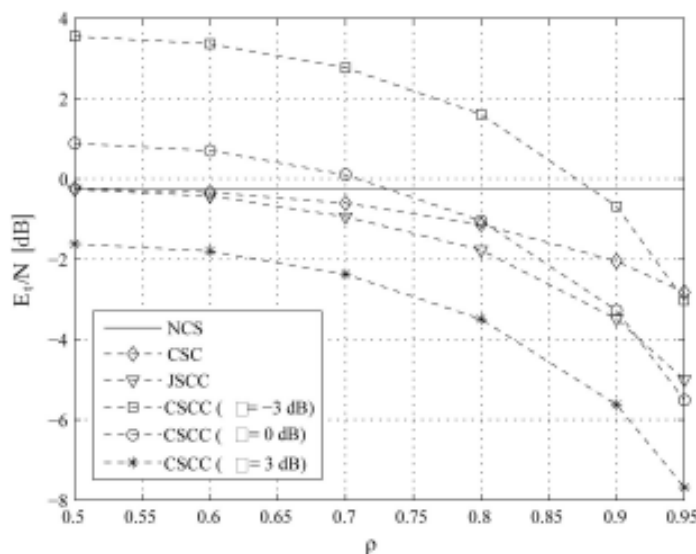


Figure 4. The average transmitted SNR, E_t/N , as a function of the source correlation coefficient, ρ , required to achieve $Pr_E = 0.01$. All four approaches are described in Sections 3.1-3.4. In all cases, $\Gamma_x = \Gamma_y = 10$ dB.

direct link, the cooperative source-channel coding scheme always outperforms the other schemes, even for very low values of the correlation coefficient ρ . Moreover, for highly correlated sources ($\rho \rightarrow 1$), the cooperative source-channel coding scheme always outperforms the non-cooperative system scheme by 14 dB, and the cooperative source-channel coding scheme by 10 dB, results that clearly show the potential benefits of cooperation. For $\theta < 0$ dB, i.e., when the SNR in the indirect link is lower than that in the direct link, then the performance of the cooperative source-channel coding scheme degrades remarkably. This is intuitive, since in this case the two sensors cannot reliably exchange information, and cooperation among them might cause even more errors (each sensor receives a “wrong suggestion” from the other one). It is worth noting that when $\rho \rightarrow 0$ (independent information signals), non-cooperative system and cooperative source-coding schemes are equivalent, and the cooperative source-channel coding scheme becomes a cooperative diversity scheme, similar to those already investigated in the literature (e.g., see [2-6]). In this case, if the channels from the nodes to the access point are the same as the channel between the nodes (i.e., $\theta = 0$ dB in our scenario), cooperation is not useful, and the non-cooperative system scheme slightly outperforms the cooperative source-channel coding scheme.

From the results shown in Figure 4, the following simple rules for the selection of the best scheme to be used, can be summarized as follows:

- If the sensors cannot cooperate with each other, then the joint source-channel coding scheme is to be preferred with respect to the cooperative source coding scheme (and, obviously, to the non-cooperative system): this is also motivated in this case by the impossibility of performing link adaptation.
- If the sensors can cooperate with each other and ϑ is sufficiently higher than 0 dB (i.e., the inter-sensor links have better quality than the direct links), then the cooperative source-channel coding scheme is to be preferred with respect to the joint source-channel coding scheme, regardless of the value of ρ .

- If the sensors can cooperate with each other but ϑ is lower than 0 dB (i.e., the inter-sensor links have worse quality than the direct links), then the cooperative source-channel coding scheme’s performance degrades significantly, and the joint source-channel coding scheme is to be preferred, regardless of the value of ρ .
- If the sensors can cooperate with each other and ϑ is around 0 dB (i.e., the inter-sensor links have a quality approximately equal to that of the direct links), then the cooperative source-channel coding scheme is to be preferred only for very high values of ρ . Otherwise, the joint source-channel coding scheme is the way to go.

Although a cooperative source-channel coding scheme may guarantee a significant advantage with respect to a joint source-channel coding scheme (without cooperation), our results do not prove that considering separate source-channel coding is the best approach in the presence of cooperation between the nodes. Determining the “optimal” approach in the presence of cooperation is an interesting research direction.

4. A Practical Joint Source-Channel Coding Scheme with Distributed LDPC Coding

In this section, we propose a practical joint source-channel coding (or JCD) scheme based on the use of low-density parity-check codes. While the structure of the iterative decoder at the access point was originally proposed in a scenario with additive-white-Gaussian-noise direct links and two sources in [25, 26], this scheme has been extended to a scenario with a generic number of correlated sources and block-faded channels [24]. A scenario of this type, with M sources, is shown in Figure 5.

The information sequences are separately encoded using identical low-density parity-check codes and transmitted over the communication links. The common coding rate at the sources is $r = 1/2$. The proposed iterative

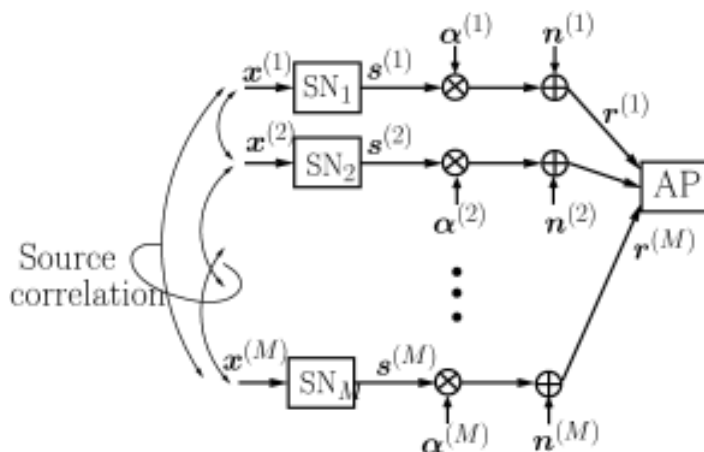


Figure 5. A scheme with a generic number of correlated sources and block-faded channels.

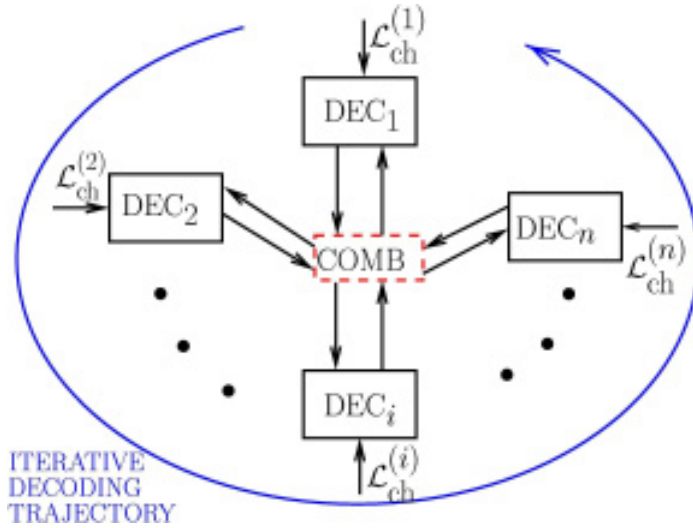


Figure 6. An iterative decoding scheme of correlated data in the absence of relay.

Each component decoder,

$DEC_i, (i = 1, \dots, n)$, is a low-density parity-check decoder that receives both the channel a posteriori reliabilities and the a priori probabilities obtained by properly processing the soft-output reliability values generated by the other decoders. These processing/combining operations are carried out in the central block denoted as “COMB.”

decoding scheme at the access point is shown in Figure 6, where a low-density parity-check decoder per source is considered, and the trajectory of the iterative decoding process among these source decoders is highlighted: this decoding scheme is an extension of those discussed relative to two sources in [21]. Each low-density parity-checked sequence is decoded by using the classical sum-product algorithm [27]. Under the assumption of perfect channel-state information (CSI) at the receiver, the channel log-likelihood ratio (LLR) at the input of the i th variable node [27] can be expressed as

$$\mathcal{L}_{i,\text{ch}}^{(k)} = \ln \frac{P[r_i^{(k)} | s_i^{(k)} = 1, \alpha_i^{(k)}]}{P[r_i^{(k)} | s_i^{(k)} = -1, \alpha_i^{(k)}]} = \frac{2r_i^{(k)} \sqrt{E_c^{(k)}} |\alpha_i^{(k)}|}{\sigma^2}, \quad (1)$$

where $\sigma^2 = N_0/2$ and N_0 is the variance of the additive white Gaussian noise sample. The maximum number of internal decoding iterations in each component low-density parity-check decoder is denoted as $n_{\text{it}}^{\text{int-max}}$.

The a priori information about the correlation between the sources is exploited by applying the following external iterative decoding steps between the component low-density parity-check decoders: (i) the a posteriori reliability (i.e., the LLR) on the information bits of the j th decoder is properly modified, taking into account the correlation (as will be explained later), and used as a priori reliability for the information bits at the input of the ℓ th decoder ($j \neq \ell$); (ii) at the first external iteration, the a priori reliability on the information bits at the input of the ℓ th decoder is obtained by properly modifying the a posteriori reliability of the j th decoder ($j < \ell$); (iii) the algorithm stops when a maximum number of external iterations (denoted as $n_{\text{it}}^{\text{ext}}$) is reached.

The total a posteriori reliability at the input of each variable node of the factor graph underlying the ℓ th low-density parity-check decoder can be expressed as follows:

$$\mathcal{L}_{i,\text{in}}^{(\ell)} = \begin{cases} \mathcal{L}_{i,\text{ch}}^{(\ell)} + \mathcal{L}_{i,\text{ap}}^{(\ell)}, & i = 0, \dots, N/2 - 1 \\ \mathcal{L}_{i,\text{ch}}^{(\ell)}, & i = N/2, \dots, N - 1 \end{cases}.$$

In other words, besides the channel reliability value expressed as in Equation (1), the a posteriori reliability at the input of the variable nodes associated with the information bits ($i = 0, \dots, N/2 - 1$) includes the “suggestion” (given by the soft reliability value $\mathcal{L}_{i,\text{ap}}^{(\ell)}$) obtained from a posteriori reliability values output by the other decoders. In particular, the a priori component of the a posteriori reliability at the input of the ℓ th decoder can be written as

$$\mathcal{L}_{i,\text{ap}}^{(\ell)} = \ln \frac{P[s_i^{(\ell)} = 1]}{P[s_i^{(\ell)} = -1]}, \quad i = 0, \dots, L - 1,$$

where

$P[s_i^{(\ell)} = \pm 1]$ are derived from the soft-output values generated by the other decoders, as follows. In a straightforward manner, one can rewrite $P[s_i^{(\ell)}]$ as

$$P[s_i^{(\ell)}] = \frac{1}{M-1} \underbrace{\{P[s_i^{(\ell)}] + \dots + P[s_i^{(\ell)}]\}}_{M-1 \text{ times}}. \quad (2)$$

Using Bayes’ theorem [28], the probability $P[s_i^{(\ell)}]$ can be expressed as

$$P[s_i^{(\ell)}] = \sum_{s_i^{(k)} = \pm 1} P[s_i^{(\ell)}, s_i^{(k)}]$$

$$= \sum_{s_i^{(k)} = \pm 1} P[s_i^{(\ell)} | s_i^{(k)}] P[s_i^{(k)}], \quad (3)$$

$k = 1, \dots, N$ and $k \neq \ell$.

Approximating the a priori probability $P[s_i^{(k)}]$ in Equation (3) with the a posteriori reliability value, denoted as $\hat{P}[s_i^{(k)}]$, output by the k th decoder ($k \neq \ell$), from Equation (3) one obtains

$$P[s_i^{(\ell)}] \approx \sum_{s_i^{(k)} = \pm 1} P[s_i^{(\ell)} | s_i^{(k)}] \hat{P}[s_i^{(k)}]$$

where

$$\hat{P}[s_i^{(k)}] = \begin{cases} \frac{e^{\mathcal{L}_{i,\text{out}}^{(k)}}}{1 + e^{\mathcal{L}_{i,\text{out}}^{(k)}}} & \text{if } s_i^{(k)} = +1 \\ \frac{1}{1 + e^{\mathcal{L}_{i,\text{out}}^{(k)}}} & \text{if } s_i^{(k)} = -1 \end{cases}$$

At this point, we evaluate the conditional probability $P[s_i^{(\ell)} | s_i^{(k)}]$ in Equation (3) using the *a priori* distribution (rather than a posteriori reliability values). By using Bayes' theorem, it follows that

$$P[s_i^{(\ell)} | s_i^{(k)}] = \frac{P[s_i^{(\ell)}, s_i^{(k)}]}{P[s_i^{(k)}]}$$

$$= 2P[s_i^{(\ell)}, s_i^{(k)}],$$

where we have used the fact that $P[s_i^{(k)} = -1] = P[s_i^{(k)} = +1] = 1/2$, since the BPSK symbols are supposed to be *a priori* equi-probable. Finally, Equation (2) can be approximated as

$$P[s_i^{(\ell)}] = \frac{1}{M-1} \left\{ \sum_{s_i^{(1)} = \pm 1} P[s_i^{(\ell)}, s_i^{(1)}] \right.$$

$$+ \dots + \sum_{s_i^{(\ell-1)} = \pm 1} P[s_i^{(\ell)}, s_i^{(\ell-1)}]$$

$$\left. + \sum_{s_i^{(\ell+1)} = \pm 1} P[s_i^{(\ell)}, s_i^{(\ell+1)}] + \dots + \sum_{s_i^{(n)} = \pm 1} P[s_i^{(\ell)}, s_i^{(n)}] \right\}$$

$$\approx \frac{2}{M-1} \sum_{\substack{k=1 \\ k \neq \ell}}^M \sum_{s_i^{(k)} = \pm 1} \underbrace{\hat{P}[s_i^{(k)}]}_{[\text{from decoder } k]} \underbrace{P[s_i^{(k)}, s_i^{(\ell)}]}_{[\text{a priori source correl.}]}$$

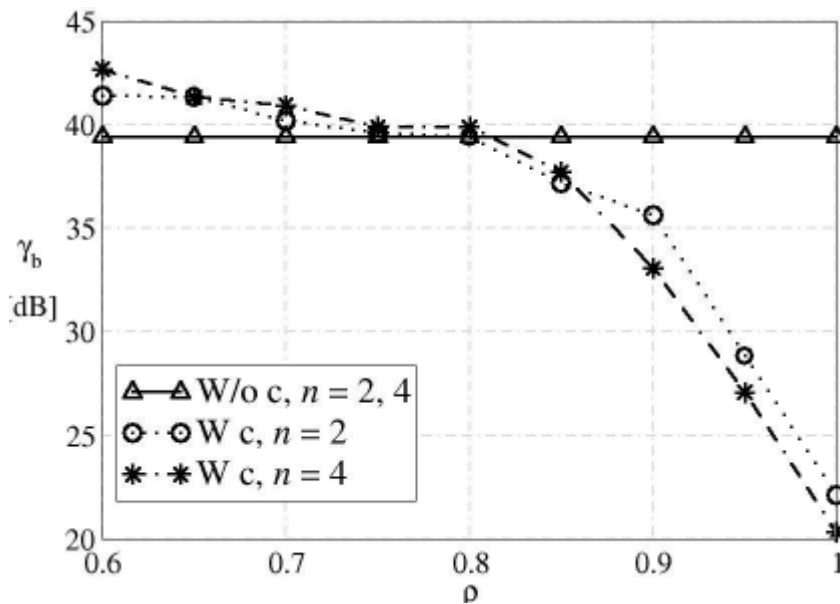


Figure 7. The SNR as a function of the correlation coefficient, ρ , required to achieve a BER equal to 10^{-4} in a low-density parity-check coded scenario with block-faded links. The number of sources, n , is set to either two or four. For comparison, the SNR required when the source correlation is not exploited at the access point (without c) is also shown.

where $P[s_i^{(k)}, s_i^{(\ell)}]$ can be obtained by marginalization of the n th dimensional a priori joint PMF $\left\{P[s_i^{(1)}, s_i^{(2)}, \dots, s_i^{(M)}]\right\}$ of the information sequences at the input of the sources⁵. The intuition behind Equation (4) consists in modifying the input a priori probability of a single bit by taking into account – through a weighed average – the reliability values (on the same bit) generated by the other decoders. In particular, the weight of the reliability value generated by the k th decoder is given by the joint a priori probability between the ℓ th and the k th decoders.

We now consider a specific scenario where each of the source sequences is encoded using a regular (3,6) low-density parity-check code with rate 1/2 and $N = 2000$. Each component decoder performs a maximum number $n_{it}^{int-max}$ of internal iterations, set to 50, whereas the number n_{it}^{ext} of external iterations between the two decoders is set to 20. The low-density parity-check code is constructed in a *random* fashion, according to the following algorithm, which exploits an idea similar to the progressive-edge-growth (PEG) algorithm presented in [29]. Some potential connections, denoted as *sockets*, are drawn for all the variable and check nodes. For each variable node, a socket is then randomly chosen among all the free sockets at the check nodes, and the connection is added only if a *cycle* of a given (or lower) length is not created. In our case, the checked cycle length is equal to six.

In Figure 7, the SNR required to achieve a bit error rate (BER) equal to 10^{-4} is shown, as a function of the *correlation coefficient*, ρ , in various low-density parity-check-coded scenarios. In the relayed cases, the source-relay links are ideal. The number of sources, n , is either two or four. For comparison, the SNR required when the source correlation is not exploited is also shown. While for values of ρ lower than 0.8, exploiting the correlation leads to a (limited) SNR loss; for higher values of ρ , the SNR gain becomes significant. At very low values of ρ , the SNR loss is due to the fact that the iterative decoding scheme at the access point does not converge. In other words, the suggestions that the decoders pass to each other are not reliable and degrade the performance. However, the number of sources seems to have a limited impact on the SNR gain.

5. Concluding Remarks

In this paper, we have presented a perspective on cooperative coding for distributed radio systems. In particular, in a scenario with correlated sources, four possible approaches have been analyzed to exploit the source correlation: a non-cooperative system, cooperative source coding, cooperative source-channel coding, and joint source-channel coding. The differences between these approaches reside on how source correlation is exploited: either at the sources and/or at the access point. Obviously, the system complexity concentrates at the system position where

correlation is exploited. In all approaches considered, we have derived the probability of error, i.e., the probability of incorrect delivery of the sources' information signals, and a comparative analysis has been carried out. In the joint source-channel coding case, a few idealistic assumptions were considered: ideal power control and ideal channel coding. However, it has been observed that joint source-channel coding systems are very attractive in wireless scenarios where the communication channels between the sources may be very noisy. As an illustrative example of the feasibility of these schemes, we have considered a low-density parity-check-coded joint source-channel coding system in a scenario with block-faded direct links and an arbitrary number of sources.

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¹ In [16], the communication links are affected by additive white Gaussian noise, whereas in this work we extend this approach to encompass the presence of fading.

² Note that the ultimate feasibility region, obtained in the presence of faded channels with ideal power control, coincides with that in the presence of additive white Gaussian noise links.

³ Note that only the information bits are considered in the exchange of reliability information between the component low-density parity-check decoders, since the coded bits are not directly correlated.

⁴ The joint PMF of $\{s_i^{(k)}\}^M$ can then be obtained directly from the correlation between the information sequences. Note that Equation (4) is an approximation, since heuristically the first probability in the summation on the right-hand side is obtained from the reliability values generated by the other decoder, whereas the second probability is a priori.