A Combinatorial Approach to the Optimization of the Channel Utilization Ratio in Ad Hoc Wireless Networks

G. Ferrari*[,1] and O.K. Tonguz[2]

[1]Department of Information Engineering, University of Parma, Parma, Italy

[2]Department of Electrical and Computer Engineering, Carnegie Mellon University, Pittsburgh, PA, USA

Abstract: In this paper, after formalizing the concept of *channel utilization ratio* (CUR) in ad hoc wireless networks, we investigate its impact on the network performance. Given that a node can hold a multi-hop route for a time interval defined as *reserved channel utilization interval* (RCUI), we assume that the node effectively utilizes the reserved route for an interval defined as *effective channel utilization interval* (ECUI), the duration of which corresponds to the duration of the message to be transmitted. This models a realistic scenario, where a node may use the shared radio medium for only a portion of the reservation interval. Defining the CUR as the ratio between the durations of the RCUI and the ECUI, we develop a combinatorial framework which leads to the optimization of the CUR for maximizing the *effective transport capacity*, which represents the actual bandwidth-distance product carried by the network. We also show that the way, either *continuous* or *discontinuous*, in which a node transmits its message entails minor performance differences, indicating clearly that the CUR is a meaningful network performance indicator.

1. INTRODUCTION

Ad hoc wireless networks represent a new communication paradigm which has been developing over the last years. An important performance measure for this type of networks is the transport capacity, an information-theoretic concept introduced in [1], which can be viewed as the theoretical maximum bandwidth-distance product that a wireless network is capable of supporting. While most of the literature on ad hoc wireless networks has concentrated on the design of medium access control (MAC) and routing protocols [2], the physical layer plays a major role in influencing the performance of ad hoc *wireless* networks. Researchers have considered, for example, the impact of the physical channel characteristics on the "optimal" node transmission radius [3, 4]. However, in ad hoc wireless networks, a *comprehensive* cross-layer design is needed to really capture the effects of the physical layer on higher layers. In [5], the author studies the impact of a Rayleigh fading channel on the design of routing protocols for ad hoc wireless networks. In [6], the impact of the physical layer is taken into account to design novel MAC protocols and study the relationship between communication-theoretic quantities and network connectivity. In particular, the concept of *effective transport capacity* is introduced to quantify the "actual" bandwidth-distance product carried by the network [6].

In this paper, we investigate the impact of the channel utilization ratio (CUR) of the nodes (assuming a common behavior for all nodes) on the performance of ad hoc wireless networks. More precisely, we investigate the relation between the CUR and the effective transport capacity. After reserving a multi-hop route for its intended destination, a node is assigned a specific time interval to hold this route. This time interval is defined as *reserved channel utilization interval* (RCUI), during which the node has to transmit the desired message. The time *effectively* needed to transmit this information, i.e., the time duration of a message, is defined as *effective channel utilization interval* (ECUI). The ratio between the durations of the ECUI and the RCUI corresponds to the CUR. We show that, given a particular network communication scenario, the CUR can be optimized to maximize the effective transport capacity. We introduce a simple discrete-time network communication model, where the basic time unit is the bit duration. Hence, the RCUI and ECUI durations can be equivalently specified as integer values, and this allows to resort to a novel *combinatorial analysis* of the inter-node interference (INI). In particular, two possible transmission strategies will be considered: (i) *continuous* transmission, such that the message stored at each node is transmitted continuously for its entire duration, and (ii) discontinuous (or *randomized*) transmission, such that the message is transmitted in "random pieces" within the RCUI. Ideal (without INI) and realistic (with INI) network communication scenarios are considered. It is shown that the specific transmission strategy, either continuous or randomized, entails minor performance differences. We finally comment on the relation between the obtained analytical results and the simulation results presented in [7, 8]. We preliminary underline that the considered network communication model will be deliberately simple, in order to derive, in a relatively tractable analytical way, meaningful insights into this interesting problem.

This paper is structured as follows. In Section 2, we summarize basic assumptions behind the considered ad hoc wireless networking scenario. The problem statement is given in Section 3. Preliminaries on topology, RCUI/ECUI

*Address correspondence to this author at the Department of Information Engineering, University of Parma, Parma, Italy;
E-mail: gianluigi.ferrari@unipr.it

1874-1290/07

concepts, and effective transport capacity are provided in Section 4, in order to make accessible for the reader the performance analysis in Section 5. Numerical results are presented in Section 6 and discussed in Section 7. Finally, conclusions are drawn in Section 8. In Appendix A and Appendix B, the probability of bit interference from a single route and the single route interference power are evaluated, respectively.

2. BASIC ASSUMPTIONS

In this section, the main assumptions behind the network communication model considered in the remainder of this paper are summarized.

- Before a node starts transmitting data to a desired destination, it must reserve a multi-hop route. After route reservation,[1] a node can hold the route for an RCUI, the duration of which will be indicated in the following as T_{RCUI}.
- Buffering is not explicitely considered. In other words, we are implicitly assuming that storing capabilities of the nodes are unlimited.
- We assume that active multi-hop routes are *disjoint*. This simplifies the derivation of the effective transport capacity. However, the proposed approach can be extended to the case of *crossing* routes as well. In such a scenario, the queuing strategy at each node comes into play [9].
- A relay node in a multi-hop route cannot transmit its own message while the source has not released the route.
- We assume that each node generates information at a *constant* bit-rate λ_{b} (dimension: [b/s]). While this is somewhat unrealistic, it allows derivation of closed-form expressions which can also be applied, as guidelines, to scenarios where information is not generated at a constant rate.
- The transmission bit rate, indicated as R_{b} (dimension: [b/s]), is the same for all nodes.
- The maximum number of routes that can simultaneously be active will be indicated as $N_{\mathrm{R}}^{\max}$. In general, $\nu \leq N_{\mathrm{R}}^{\max}$ routes are active in the network. Note that the number of simultaneously active routes in the network depends on the traffic nature at each node and on the particular "history" of routes' activation. This aspect is beyond the scope of this paper, where we are interested in characterizing the network communication performance at regime.
- For analytical purposes, perfect synchronization among the active nodes is considered. In other words, RCUIs of active source nodes are synchronous. Every T_{RCUI}, ν routes become active. This is shown in Fig. (**1**), in a simple case with ν=3 routes active at a time. As one can see, in the first RCUI, three nodes, indicated as n_1,n_2, and n_3, transmit to their respective destinations, indicated as d_1,d_2, and d_3—note that each node, within the RCUI, transmits for an ECUI, the duration of which could be shorter than that of the RCUI. In the following RCUI, three other nodes, indicated as n_4,n_5, and n_6, transmit to their respective destinations, and so on until the (N/ν-1)-th RCUI, during which the last three nodes, i.e., n_{N-2},n_{N-1}, and n_N, transmit. From the N/ν-th RCUI on, the nodes' transmission sequence starts over again.[2] This scenario refers to the case of *ideal fairness*. More precisely, in a network communication scenario with ν active routes at a time, indicating by N the total number of nodes in the network, the time between two successive transmission acts by the same node is $(N/\nu)T_{\mathrm{R}CUI}$.

  We point out, however, that the final result, in terms of the existence of an optimal CUR, holds also if (i) there is no synchronization among the RCUIs of simultaneously active nodes and/or (ii) there is no ideal fairness, in the sense that a node's RCUI duration is longer than that of the other nodes, *provided that* the CUR is the same for all nodes. The validity of the obtained results will be discussed in more detail in Section 7.
- A message to be transmitted by a source node, after route reservation, is formed by a fixed number, indicated as k, of bits.
- The nodes are assumed to be *static* and placed at the vertices of a *regular square grid*: each node has four nearest neighbors (at the same distance). This topology is shown in Fig. (**2**). The extension of the proposed analysis of the optimal CUR to scenarios with *mobile nodes* and/or *random topology* can be considered by following the approach proposed in [10, 11].

3. PROBLEM STATEMENT

Assume that a source node, after reserving a multi-hop route to its destination, holds it for a time interval of duration T_{RCUI}. The key question addressed in this paper is the following: given that the effective transmission lasts for an interval of duration T_{ECUI} (with $T_{\mathrm{ECUI}} \leq T_{\mathrm{RCUI}}$), is there an "optimized" value of the CUR (i.e., an optimized value of T_{ECUI}) such that the effective transport capacity is maxi-

[1]Route reservation is assumed to be successfully accomplished, and our analysis focuses only on the transmission phase.

[2]Note that the destination nodes might change from an activation of a group of ν nodes to the next activation of the same nodes, as shown in Fig. (**1**) for nodes n_1, n_2, and n_3.

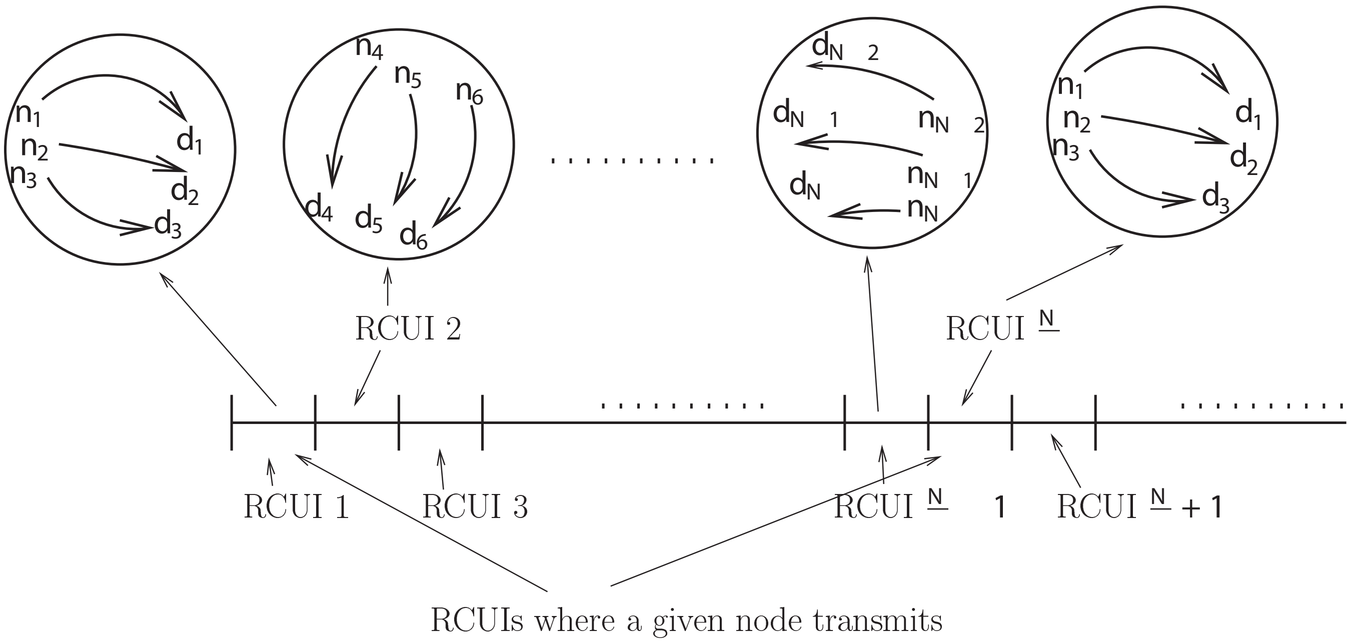

Fig. (1). A pictorial description of a perfectly *synchronized* and *fair* ad hoc wireless network communication scenario with *disjoint* routes. In each RCUI, ν=3 routes are active. In consecutive RCUIs different groups of routes become active, and the activation schedule resumes every *N*/ν RCUIs.

mized? Considering the wireless network communication scenario characterized by the assumptions outlined in Section 2, we will be able to answer this question.

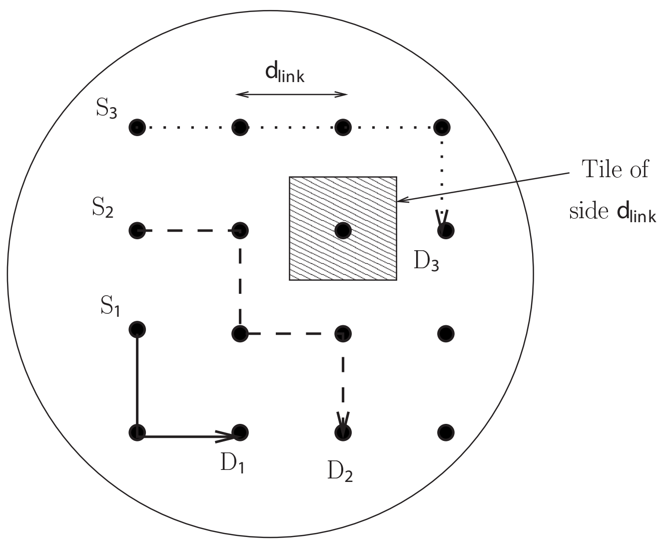

Fig. (2). Multi-hop network communication scenario in an ad hoc wireless network with square grid topology and circular surface.

A similar problem has been considered in [7], in the case of large and dense packet radio networks, and in [8] in the case of non-reservation-based radio packet networks based on an IEEE 802.11-type wireless network communication model [12]. In order to reduce the collisions between ongoing communications, in [7] a common randomized transmission scheduling between communicating nodes is considered, whereas in [8] a pseudorandom time slot choice, based on the generation of a random seed exchanged between neighboring nodes, is used. We will show how our analytical results compare to those, based on the use of computer simulations, presented in [7,8]. Insights into the relationship between CUR and network connectivity will also be provided, and it will be shown that they are in good agreement with recent results obtained with the use of percolation theory [13].

4. PRELIMINARIES

In this section, on the basis of the considered assumptions in Section 2, we give preliminaries on the characteristics of the considered ad hoc wireless network communication scenario. In particular, we first comment on the routes distribution in the network. Then, a simple expression for the bit error rate (BER) at the end of a multi-hop route is derived and a necessary condition for network operation is proposed, based on the definitions of RCUI and ECUI. Finally, an expression for the effective transport capacity in the considered ad hoc wireless networking scenario is provided.

4.1. Topology and Routes

We assume that a multi-hop route is constituted by a sequence of minimum-length hops (i.e., a node communicates directly only to one of its four neighboring nodes). In Fig. (**2**), three possible disjoint multi-hop routes are indicated. Considering *N* nodes over a circular surface with area *A*, it is possible to show that the distance between two neighboring nodes, indicated by d_{link}, can be written (neglecting border effects) as [6]

$$d_{\text{link}} = \frac{1}{\sqrt{\rho_S}}$$

where $\rho_S \triangleq N/A$ is the node spatial density. Indicating by $\overline{n}_h$ the average number of hops in a route (an expression for which is provided in the following subsection), there can thus be, at most, $N_{\mathrm{R}}^{\max} \triangleq N/\overline{n}_{\mathrm{h}}$ disjoint routes simultaneously active in the network. In general, as observed in Section 3, there may be $\nu \leq N_{\mathrm{R}}^{\max}$ simultaneously active multi-hop routes, depending on the network communication "history."

4.2. Route Bit Error Rate

Denoting by $\mathrm{BER}_{\mathrm{link}}$ the BER at the end of a single link, and assuming that (i) each link (of length d_{link}) is characterized by the same BER,[3] (ii) there are no burst errors, and (iii) the uncorrected bit errors in a link (after the processing at the receiving node of the link) are never recovered from in the following links, it is possible to show that the BER at the end of a multi-hop route with n_{h} hops, indicated as $\mathrm{BER}_{\mathrm{route}}^{(n_{\mathrm{h}})}$, can be expressed as[4] [6]

$$\mathrm{BER}_{\mathrm{route}}^{(n_{\mathrm{h}})} = 1-(1-\mathrm{BER}_{\mathrm{link}})^{n_{\mathrm{h}}}. \tag{1}$$

Expression (1) shows the dependence of the BER at the end of a multi-hop route on the number of hops n_{h} and on the link BER. In order to make an *average* network performance analysis, we evaluate the BER at the end of a multi-hop route with an *average* number of hops. In other words, we evaluate the BER expression (1) in correspondence to an average number of hops $\overline{n}_{\mathrm{h}}$. Therefore, we now derive a simple expression for the average value $\overline{n}_{\mathrm{h}}$. It is possible to show that a good statistical description of the number of hops n_{h} is given by a "quasi-binomial" distribution, obtained from a binomial distribution by eliminating the probability mass concentrated in zero and renormalizing the other probability masses—this is intuitively justified by the fact that very short or very long routes are more unlikely than routes with intermediate length. We therefore assume that the number of hops n_{h} is quasi-binomially distributed between one and the maximum number of hops, indicated as $n_{\mathrm{h}}^{\max}$, over a diameter of the circular network surface. It can be shown that $n_{\mathrm{h}}^{\max} = 2r_A/d_{\mathrm{link}} = \lfloor 2\sqrt{N/\pi} \rceil$, where r_A is the network circular surface radius and the notation $\lfloor * \rceil$ indicates the integer value closest to the argument $*$ [6]. Therefore, one obtains $\overline{n}_{\mathrm{h}} = n_{\mathrm{h}}^{\max}/2 = \lfloor \sqrt{N/\pi} \rceil$. . The BER at the end of a multi-hop route with an average number of hops can then be written as follows:

$$\mathrm{BER}_{\mathrm{route}} = \mathrm{BER}_{\mathrm{route}}^{(\overline{n}_{\mathrm{h}})} = 1-(1-\mathrm{BER}_{\mathrm{link}})^{\overline{n}_{\mathrm{h}}}.$$

We observe that considering all possible source/destination pairs and evaluating the average route length, it turns out that the "true" average value is slightly lower than $n_{\mathrm{h}}^{\max}/2$. In general, however, regardless of the surface shape, it is possible to show that $\overline{n}_{\mathrm{h}} = \Theta(\sqrt{N})$, where the notation $\Theta(x)$ denotes a quantity "on the order of" x [14].

In order to compute the route BER, the link BER has to be computed. In [6], through a rigourous detection-theoretic approach the link BER is computed and it is shown that the Gaussian assumption for the interference process does not generally hold. More precisely, it can be shown that there exists a BER "floor" which depends on the MAC protocol under use. *Provided that* the maximum tolerable BER at the end of a multi-hop route is higher than the floor route BER (this is the case in typical operative conditions), the Gaussian assumption for the interference noise becomes accurate [6]. In this case, in the presence of binary modulation, the link SNR can be written as follows:

$$\mathrm{SNR}_{\mathrm{link}} = \frac{P_{\mathrm{r}}}{P_{\mathrm{thermal}} + P_{\mathrm{int}}} \tag{2}$$

where P_{r} is the received power, P_{thermal} is the thermal noise power, and P_{int} is the interference noise power. In the remainder of this paper we will assume uncoded binary phase shift keying (BPSK) signaling. Under the Gaussian assumption for the overall noise process, the link BER can be written as

$$\mathrm{BER}_{\mathrm{link}} = Q\left(\sqrt{2\,\mathrm{SNR}_{\mathrm{link}}}\right)$$

where $Q(x) \triangleq \int_{\infty}^{x} \frac{1}{\sqrt{2\pi}} e^{-u^2/2}\, \mathrm{d}u$. Note that the extension of the proposed analysis to other channel models is straightforward, provided that the appropriate link BER expression is used [15]. We now characterize the three powers appearing in the link SNR expression (2).

- We assume that the transmitted signal is affected by free-space loss. Hence, according to Friis formula [16], the received signal power at distance d_{link} from the transmitter, indicated by P_{t}, has the following expression:

$$P_{\mathrm{t}} = \alpha \frac{P_{\mathrm{t}}}{d_{\mathrm{link}}^2} = \frac{G_{\mathrm{t}} G_{\mathrm{t}} c^2}{(4\pi)^2 f_l f_c^2} \frac{P_{\mathrm{t}}}{d_{\mathrm{link}}^2}$$

where: P_{t} (dimension: [W]) is the transmit power (common for all nodes); G_{t} and G_{t} are the transmitter and receiver antenna gains; f_{c} (dimension: [Hz]) is the

[3]The first assumption might not be exactly true in a real communication scenario affected by INI, since the level of interference experienced by a node depends on its position with respect to the other nodes. However, the obtained results will provide, trend-wise, useful insights into the impact of node mobility also in more realistic scenarios.

[4]We are implicitly assuming that the link BER is the same for all links. In a realistic communication scenario affected by interference, this is imprecise, since the BER depends on the position of the link in the network. An average interference analysis will be considered in the following, distinguishing between best-case and worst-case interference scenarios.

carrier frequency; c is the speed of light; and $f_l \geq 1$ is a loss factor.

- The thermal noise power can be written as $P_{\text{thermal}}=FkT_0B$, where F is the noise figure, k is the Boltzmann's constant, T_0=300 K is ambient temperature, and B is the transmission bandwidth, which, in the case of BPSK signaling, coincides with the transmission bit rate R_b.
- The interference power P_{int} depends, besides on the node geometry, on the MAC protocol. In the following section, we provide an explicit expression for P_{int} based on a novel *combinatorial analysis*. Aside, we remark that the performance in an *ideal* network communication scenario is obtained by setting P_{int}=0.

4.3. RCUI and ECUI

The assumption of ideal fairness leads to a "perfectly cyclic" network communication behavior: there are ν disjoint routes simultaneously active for an interval of duration T_{RCUI}; at the end of this interval, these routes are torn down and a new set of ν routes become active, and so on. Therefore, within an RCUI a node has to transmit the information (generated at a constant rate) accumulated in the previous (N/ν-1) RCUIs and the information which is being generated in the current RCUI. Hence, a node has to transmit the information which has been generated during N/ν intervals of duration T_{RCUI}. According to the assumption, introduced in Section 2, of bit generation with constant rate equal to λ_b, indicating by k the amount of bits to be transmitted, it follows that $k = \lambda_b T_{\text{RCUI}}(N/\nu)$. . In the absence of perfect transmission cyclicity among the nodes (i.e., non-ideal fairness), the analysis presented in the following would still hold, assuming that the size of a message to be transmitted by an active node was fixed—in this case, however, the assumption of constant bit-rate generation should be relaxed. In other words, imposing that the message size is fixed to k bits, if a node has to wait j RCUIs before being able to transmit, then this would correspond to an equivalent constant bit rate $\lambda_b' = \lambda_b / j$.

Considering a discrete-time model with basic time unit given by the bit duration $1/R_b$, an RCUI corresponds to $n \triangleq R_b T_{\text{RCUI}}$ time units. The message with k bits has to be transmitted within an RCUI, i.e., the condition $n \geq k$ has to be satisfied. This condition can be equivalently reformulated as

$$\frac{R_b}{\lambda_b} \geq \frac{N}{\nu}. \qquad (3)$$

If condition (3) is not satisfied, then a node cannot transmit the entire message in an RCUI.

Based on condition (3), it follows that within the RCUI a node transmits (at a data-rate R_b) for an ECUI of duration

$$T_{\text{ECUI}} \doteq \frac{k}{R_b} = \frac{\lambda_b}{R_b}\frac{N}{\nu} T_{\text{RCUI}} \qquad (4)$$

from which one obtains the following expression for the CUR:

$$\text{CUR} = \frac{T_{\text{ECUI}}}{T_{\text{RCUI}}} = \frac{k}{n} = \frac{\lambda_b}{R_b}\frac{N}{\nu}. \qquad (5)$$

Based on (4), condition (3) can be equivalently rewritten as $T_{\text{ECUI}} \leq T_{\text{RCUI}}$ or CUR<1.

4.4. Effective Transport Capacity

As a useful indicator of network connectivity we consider the *average sustainable number of hops*, which is defined as follows [6]:

$$\overline{n}_{\text{sh}} \triangleq \min\left\{ \left\lfloor \frac{\ln(1-\text{BER}_{\text{route}}^{\max})}{\ln(1-\text{BER}_{\text{link}})} \right\rceil, \overline{n}_{\text{h}} \right\}$$

where $\text{BER}_{\text{route}}^{\max}$ is the maximum acceptable BER at the end of a multi-hop communication route (this will be the physical layer-based quality of service, QoS, considered in this paper). In other words, the average sustainable number of hops is the minimum between the maximum sustainable number of hops (which can be written as $\lfloor \ln(1-\text{BER}_{\text{route}}^{\max})/\ln(1-\text{BER}_{\text{link}}) \rceil$) and the average number of hops $\overline{n}_{\text{h}}$. Recalling that the hop length is $d_{\text{link}}=1/\sqrt{\rho_S}$, the average path length $\overline{d}_{\text{path}}$ can thus be written as

$$\overline{d}_{\text{path}} \triangleq \overline{n}_{\text{sh}}\, d_{\text{link}} = \overline{n}_{\text{sh}}\sqrt{\frac{A}{N}}.$$

At this point, we introduce the *effective transport capacity*, representing the *actual* bandwidth-distance product that can be sustained by the network [6]. This quantity is obtained by combining the contributions from the various active routes, each of which carries an "effective" data-rate equal to $(T_{\text{ECUI}}/T_{\text{RCUI}})R_b$=CUR$R_b$ over a path with average length equal to $\overline{d}_{\text{path}}$. The effective transport capacity associated with a single route, indicated as $C_{\text{Te}}^{(\text{sr})}$, can therefore be written as

$$C_{\text{Te}}^{(\text{sr})} = \text{CUR}\ \underbrace{R_b}_{[b/s]}\ \underbrace{\overline{d}_{\text{path}}}_{[m]}.$$

Given that there are ν active routes, the aggregate effective transport capacity is

$$C_{\text{Te}} \doteq \nu\, C_{\text{Te}}^{(\text{sr})} = \nu\, \text{CUR}\, R_b\, \overline{d}_{\text{path}}. \qquad (6)$$

5. PERFORMANCE ANALYSIS

In this section, based on the ad hoc wireless network communication scenario introduced in the previous sections, we provide a performance analysis distinguishing between an ideal case, where there is no INI, and a realistic case, where communications are affected by INI.

5.1. Ideal Network Communication Scenario

An ideal wireless network scenario (where communications are not affected by INI), i.e., with P_{int}=0, would correspond to a scenario where the MAC protocol is "perfect," in the sense that a node accesses the shared radio medium without damaging any other active inter-node communication. In this case, the number ν of active routes has no influence on the interference: it only affects the effective transport capacity. It is easy to conclude that the highest effective transport capacity is obtained when a node transmits for the entire RCUI duration, i.e., when T_{ECUI}=T_{RCUI} or, equivalently, CUR=1. Intuitively, this is obvious: since there is no interference, a node can use the reserved route as much as possible. This will be confirmed by the results shown in Section 6.

5.2. Realistic Network Communication Scenario

In order to evaluate the INI power in a realistic scenario, we distinguish between two possible transmission strategies, either *continuous* or *discontinuous*, which can be adopted by a node in transmitting a *k*-bit message during its RCUI. In particular, we first derive the (average) probability of bit interference in the case of continuous transmission, then the (average) probability of bit interference in the case of discontinuous transmission, and, finally, the average interference power.

5.2.1. Probability of Bit Interference: Continuous Transmission

In order to analyze this scenario, we assume that an active source node transmits its *k*-bit message continuously in the RCUI. This transmission strategy can be depicted through the matrix model shown in Fig. (**3**): each row corresponds to an active multi-hop route, while each column indicates a time unit (i.e., a bit position) in the RCUI.

Each filled position in this matrix model corresponds to a bit "flying" from source to destination in the route corresponding to the row. The model in Fig. (**3**): is based on the implicit assumption of neglecting the propagation delay and the processing time at each intermediate node of a multi-hop route. While neglecting the propagation delay is reasonable in wireless networks, neglecting the processing time at the intermediate nodes might not be so. If one wanted to take into account the processing time, the matrix-based model in Fig. (**3**) would still hold, provided that the time unit was longer than the bit duration $1/R_b$ and took into account an average processing time at intermediate nodes. One could therefore extend straightforwardly the analysis proposed in the following.

Assuming that each active source node starts transmitting independently from the other ν-1 active source nodes in the

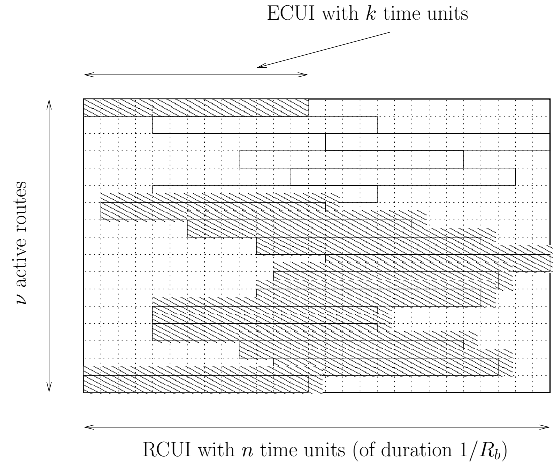

Fig. (3). ν active routes transmitting in an RCUI of duration T_{RCUI}, with *n* time units. Along each route a *continuous* message *k* time units long is "flowing". In the figure, ν=17, *n*=27, and *k*=13.

network, and considering the possible positions of the first bit of each transmitted message, a "snapshot" of the network communication scenario, as depicted in Fig. (**3**), can be modeled as the outcome of an experiment where ν numbers between 1 and (*n*-*k*+1), corresponding to the first bits of the ν messages, are extracted independently. Without loss of generality, one can restrict his/her attention to a specific multi-hop route, considering the probability of interference (from other routes) as a function of the bit position. We denote the probability of interference from a single route in the *i*-th bit position as $\mathrm{p}_{\mathrm{int,c}}^{(i)}$. At this point, one has to evaluate the probability that a bit is transmitted by a source in the *i*-th position: we indicate this probability[5] as $\mathrm{p}_{\mathrm{tx,c}}(i)$. In Appendix A, explicit expressions for $\mathrm{p}_{\mathrm{int,c}}^{(i)}$ and $\mathrm{p}_{\mathrm{tx,c}}(i)$ are derived, and it is shown that the average probability of interference can be written as

$$\overline{\mathrm{p}}_{\mathrm{int,c}} \doteq \sum_{i=1}^{n} \mathrm{p}_{\mathrm{tx,c}}(i)\mathrm{p}_{\mathrm{int,c}}^{(i)} = \begin{cases} \dfrac{-n^2-4k^2+5nk-2n+5}{3k(n-k+1)} & k \le n < 2k \\ \dfrac{-4k^2+3nk+3k+1}{3(n-k+1)^2} & n \ge 2k. \end{cases}$$

It is easy to show that $\overline{\mathrm{p}}_{\mathrm{int,c}}$ is an increasing function of the CUR=*k*/*n* and

[5]Note that the probability ensemble $\{\mathrm{p}_{\mathrm{int,c}}^{(i)}\}_{i=1}^{n}$ is *not* a probability distribution as a function of *i*, whereas $\{p\int_{\mathrm{tx,c}}(i)\}_{i=1}^{n}$ is.

$$\lim_{\mathrm{CUR}\to 0}\overline{\mathrm{p}}_{\mathrm{int,c}} = 0 \qquad \lim_{\mathrm{CUR}\to 1}\overline{\mathrm{p}}_{\mathrm{int,c}} = 1.$$

Moreover, for CUR<<1, it follows that $\overline{\mathrm{p}}_{\mathrm{int,c}} \simeq \mathrm{CUR}$.

The analysis carried out in the case of a single interfering route can be straightforwardly extended to the case of *m* interfering routes (obviously, $m \in \{1,...,\nu-1\}$). In order for a bit position in the first communication route to be interfered by *m* distinct multi-hop routes, the column (in the used matrix model) corresponding to the considered bit position has to be filled in only *m* other rows (corresponding to *m* active source nodes), while it does not have to be filled in the remaining rows. Hence, there are $\binom{\nu-1}{m}$ possible combinations with only *m* interfering routes out of (ν-1) possible ones. Indicating by $\mathrm{Pr}_{\mathrm{int,c}}(m)$ the average probability of bit interference from *m* routes, it can be concluded that the probability distribution $\{\mathrm{Pr}_{\mathrm{int,c}}(m)\}_{m=1}^{\nu-1}$ is binomial with parameter $\overline{\mathrm{p}}_{\mathrm{int,c}}$, i.e.,

$$\mathrm{Pr}^{(i)}_{\mathrm{int,c}}(m) = \binom{\nu-1}{m}\left(\mathrm{Pr}^{(i)}_{\mathrm{int,c}}\right)^{m}\left(1-\mathrm{p}^{(i)}_{\mathrm{int,c}}\right)^{\nu-1-m}$$
$$m = 1,\ldots,\nu-1.$$

5.2.2. Probability of Bit Interference: Discontinuous Transmission

In this case, whenever a source node starts transmitting in a previously created communication route, the transmission of the *k*-bit message is discontinuous (or random-like), i.e, there are "gaps" between following subgroups of bits which are transmitted. This scenario can be easily depicted through the matrix model shown in Fig. (**4**).

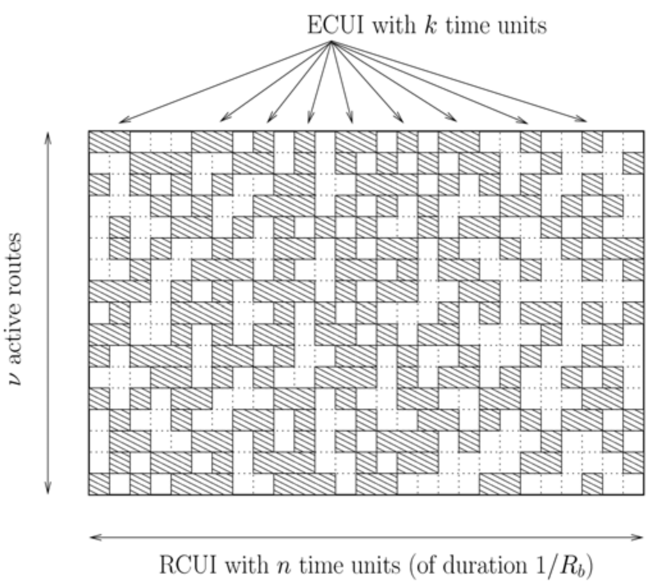

Fig. (4). ν active routes transmitting in an RCUI of duration T_{RCUI}, with *n* time units. Along each route a *discontinuous* message *k* time units long is "flowing." In the figure, ν=17, *n*=27, and *k*=13.

In Appendix A, it is shown that the probability of interference on a bit position from another single route is independent of the bit position, and can be written as

$$\mathrm{p}_{\mathrm{int,d}} = \frac{k}{n}.$$

Since it can be shown that the probability of bit transmission does not depend on the bit position (see Appendix A for more details), i.e., $\mathrm{p}_{\mathrm{tx,d}}(i) = 1/n,\ i = 1,\ldots,n,$, it follows that the average probability of bit interference from another (single) route is $\overline{\mathrm{p}}_{\mathrm{int,d}} = \mathrm{p}_{\mathrm{int,d}} = k/n.$. Therefore, in the case of discontinuous transmission, the probability of interference from *m* routes in a single bit position, indicated as $\mathrm{Pr}_{\mathrm{int,d}}(m)$, is binomial as in the case of continuous transmission, the only difference being the characteristic parameter, which is now equal to $\overline{\mathrm{p}}_{\mathrm{int,d}}$:

$$\mathrm{Pr}_{\mathrm{int,d}}(m) = \binom{\nu-1}{m}(\overline{\mathrm{p}}_{\mathrm{int,d}})^{m}(1-\overline{\mathrm{p}}_{\mathrm{int,d}})^{\nu-1-m}$$
$$m = 1,\ldots,\nu-1.$$

5.2.3. Average Interference Power

According to the final results obtained in the previous two subsections, the probability of bit interference from *m* routes, with $1 \le m \le N_R$-1, is binomial with the following parameter:

$$\overline{\mathrm{p}}_{\mathrm{int}} \triangleq \begin{cases} \overline{\mathrm{p}}_{\mathrm{int,c}} & \text{for continuous transmission} \\ \overline{\mathrm{p}}_{\mathrm{int,d}} & \text{for discontinuous transmission.} \end{cases} \qquad (7)$$

Hence, the average probability of bit interference from *m* routes, indicated as , can be written as follows:

$$\mathrm{Pr}_{\mathrm{int}}(m) = \binom{\nu-1}{m}\overline{\mathrm{p}}^{m}_{\mathrm{int}}(1-\overline{\mathrm{p}}_{\mathrm{int}})^{\nu-1-m} \qquad (8)$$
$$m = 1,\ldots,\nu-1.$$

At this point, in order to determine an expression for the average interference power in a single bit position according to the binomial distribution in (8), it is expedient to determine an expression for the average interference power, at the end of a communication link, generated by another single active route. In Appendix B, it is shown that this interference power can be bounded between the following minimum and maximum values, respectively:

$$\overline{P}^{(\mathrm{sr})-\min}_{\mathrm{int}} \triangleq \frac{\alpha P_{\mathrm{t}}\rho_{\mathrm{S}}}{i_{\max}(\overline{n}_{\mathrm{h}}+1)}\sum_{i=1}^{i_{\max}}\left(\frac{1}{i^2}+2\sum_{j=1}^{\lfloor\frac{\overline{n}_{\mathrm{h}}}{2}\rceil}\frac{1}{i^2+j^2}\right) \qquad (9)$$

$$\overline{P}^{(\mathrm{sr})-\max}_{\mathrm{int}} \triangleq \frac{\alpha P_{\mathrm{t}}\rho_{\mathrm{S}}}{i_{\max}}\sum_{i=1}^{i_{\max}}\left(\frac{1}{i^2}+2\sum_{j=1}^{\lfloor\frac{\overline{n}_{\mathrm{h}}}{2}\rceil}\frac{1}{i^2+j^2}\right) \qquad (10)$$

where $i_{\max} = \lfloor\sqrt{N}/2\rceil$ is the maximum tier order, with respect to the center, in the considered square grid node distribution. In particular, the best-case interference scenario, described by (9), corresponds to a situation where only one node of the interfering route is active, whereas the worst-case interference scenario, described by (10), corresponds to the case where all nodes of the interfering route are simultaneously active. In general, the interference power generated by another route will be a value, indicated as $\overline{P}^{(\mathrm{sr})}_{\mathrm{int}}$, between

$\overline{P}_{\rm int}^{\rm (sr)-min}$ and $\overline{P}_{\rm int}^{\rm (sr)-max}$ (depending on the number of active nodes in the interfering route). Under the hypothesis that there are m interfering active routes, we simply assume that the interference power[6] is $m\overline{P}_{\rm int}^{\rm (sr)}$. Recalling expression (8) for the probability of interference from m other routes (binomial distribution), the average interference power $\overline{P}_{\rm int}$ can be given the following expression:

$$\begin{aligned}\overline{P}_{\rm int} &= \sum_{m=1}^{\nu-1} m\overline{P}_{\rm int}^{\rm (sr)}{\rm Pr}_{\rm int}(m) \\ &= \overline{P}_{\rm int}^{\rm (sr)}(\nu-1)\,\overline{\rm p}_{\rm int}.\end{aligned} \quad (11)$$

Note that $\overline{P}_{\rm int}$ depends on the considered transmission strategy (continuous or discontinuous) through the expression of $\overline{\rm p}_{\rm int}$ given by (7).

Although the expressions for the average probability of bit interference look very different for the two cases with continuous and discontinuous transmission strategies, it turns out that the network performance is basically the same, regardless of the considered transmission strategy. At the end of Appendix A, more detailed quantitative analyses of the bit interference probabilities $\overline{\rm p}_{\rm int,c}$ and $\overline{\rm p}_{\rm int,d}$ are presented. Since $\overline{\rm p}_{\rm int,d} = k/n = {\rm CUR}$, from (11) it follows that

$$\overline{P}_{\rm int} = \overline{P}_{\rm int}^{\rm (sr)}(\nu-1)\,{\rm CUR}. \quad (12)$$

This result is intuitive, since the interference power is physically proportional to the CUR. In other words, the more the nodes use their reserved multi-hop routes, the higher is the interference.

6. OPTIMIZING THE CHANNEL UTILIZATION RATIO

Based on expression (5) for the CUR, the aggregate effective transport capacity in (6) can be rewritten as

$$C_{\rm Te} = {\rm CUR}\,\nu R_{\rm b}\sqrt{\frac{A}{N}}\,\overline{n}_{\rm sh}. \quad (13)$$

Our analysis has shown that, even in a realistic network communication scenario (both in worst-case and best-case interference scenarios), the highest effective transport capacity is obtained considering the maximum number (i.e., $\nu=N_{\rm R}^{\rm max}$) of active routes. Hence, in the following we limit ourselves to this case—the extension to a scenario with a smaller number of active routes is straightforward and the corresponding results will simply be a scaled version of those presented in the following. For $\nu = N_{\rm R}^{\rm max} = N/\overline{n}_{\rm h}$, the aggregate effective transport capacity in (13) can be equivalently rewritten as

$$C_{\rm Te} = {\rm CUR}\,R_{\rm b}\sqrt{\pi A}\,\overline{n}_{\rm sh}. \quad (14)$$

We now evaluate the effective transport capacity as a function of the CUR: the optimized' value of the CUR will correspond to the overall maximum of the effective transport capacity. The values assumed for the major network parameters are shown in Table **1**.

Table 1. Ad Hoc Wireless Network Major Parameters

Antenna gains (G_t,G_r)	1
Carrier frequency (f_c)	2.4 GHz
Noise figure (F)	6 dB
Transmitted power (P_t)	1 mW
Network area (A)	1 km^2
Number of nodes (N)	1000
Transmission data-rate (R_b)	1 Mb/s
Max. accept. route BER ($\text{BER}_{\text{route}}^{\max}$)	10^{-3}

In Fig. (**5**), the effective transport capacity is shown as a function of the CUR.

The behavior of the effective transport capacity is considered in ideal (no INI) and realistic (INI) cases. Note that this behavior does not depend on the specific message length. In fact, if the message length k is fixed, varying the CUR corresponds to varying the RCUI duration; if, instead, the RCUI duration is fixed (i.e., n is given), varying the CUR corresponds to varying the message length.

As expected, in the ideal case (i.e., when the interference power is equal to zero) the maximum effective transport capacity is proportional to the CUR (see (14)) and reaches its maximum when CUR=1. For the realistic case, two possible communication strategies are considered.

1. Each multi-hop route is assigned a *spreading code* [17]. As an example, we consider two possible values for the spreading factor, namely g=5 and g=10, and we simply assume that the interference power is reduced by a factor equal to g—this is valid, provided that the used spreading codes are distributed uniformly among the routes (which is reasonable, assuming that the number of routes is sufficiently large and each route selects a spreading code randomly[7]). Obviously, the use of per-route spreading codes requires that the available bandwidth B is sufficiently large ($B \simeq gR_{\rm b}$).

[6]The given expression of the interference power due to m routes might be incorrect for some links of the active routes. However, if the active routes are uniformly distributed in the network, this expression should be a meaningful (but pessimistic, from a spatial point of view) estimate of the interference power.

[7]In an IS-95 cellular system, the spreading factor is usually much higher than 10 [17]. Hence, the assumption of spreading factor equal to 5 in the considered ad hoc wireless network scenario should entail limited increase in processing complexity.

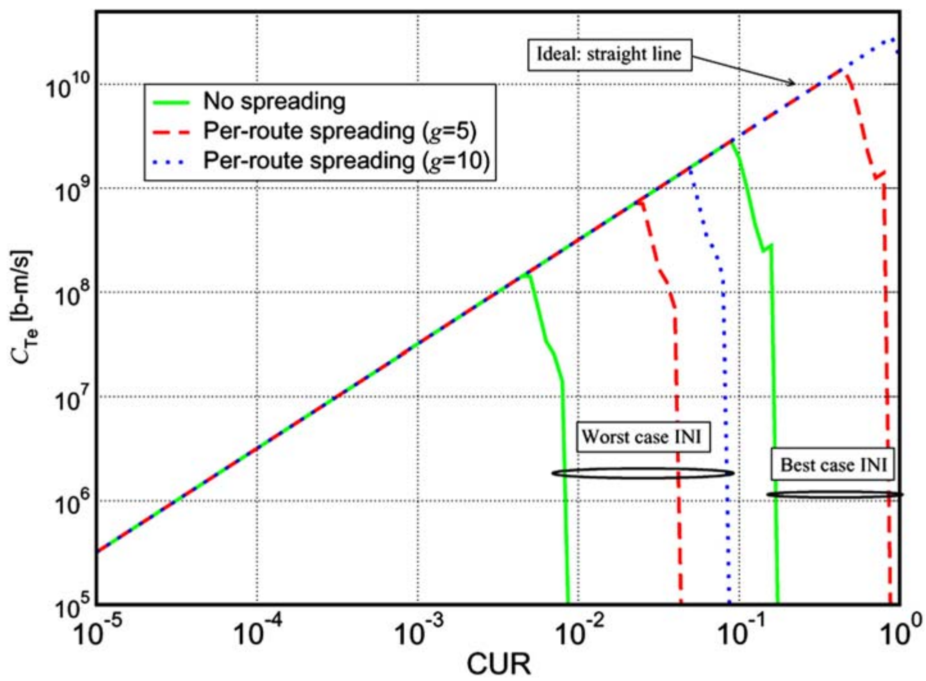

Fig. (5). Effective transport capacity as a function of the CUR. A realistic case (with best-case and worst-case INI scenarios) and the ideal (no INI) case are considered. In a realistic scenario, the presence and absence of per-route spreading codes is considered.

2. No per-route spreading code is used: all nodes in the active routes may interfere "completely" with each other.

In Fig. (**5**), for each of these communication strategies, lower and upper bounds, corresponding to worst-case and best-case interference scenarios, respectively, are shown. It is interesting to observe that, in all realistic cases, the "shape" of the curve is the same: it is proportional to the CUR up to a point, corresponding to a critical value, beyond which the maximum effective transport capacity rapidly decreases, due to an intolerable interference increase. The optimized value of the CUR is the value corresponding to the maximum of the curve.

The optimal CUR values in the realistic cases shown in Fig. (**5**) are summarized in Table **2**, distinguishing between worst-case and-best case interference scenarios.

Table 2. Crucial Values of the CUR in a Realistic Scenario, Considering Various Per-Route Spreading (PRS) Conditions

	Worst-Case Interference	Best-Case Interference
No PRS	0.48%	8.8%
PRS (*g*=5)	2.5%	44%
PRS (*g*=10)	4.9%	88%

Note that in [7,8] the CUR for optimized network performance is around 30%. Considering the results in Table **2**, and assuming that the actual networking behavior is in the middle between worst-case and best-case scenarios, it is possible to conclude that the optimal CUR in the proposed network communication scenario with disjoint routes and per-route spreading codes is around 23% if *g*=5 and about 46% if *g*=10. Therefore, considering a spreading factor between 5 and 10, one expects the average performance of our ad hoc wireless network communication scheme to be similar to that of the schemes proposed in [7,8]. While the schemes in [7,8] use scheduling mechanisms to reduce the interference, our scheme, instead, makes use of per-route spreading codes to achieve interference control. The final results, however, are in general agreement.

7. DISCUSSION

7.1. Tradeoff Between Interference Noise and Thermal Noise

In a realistic scenario, the behavior of each of the effective transport capacity curves in Fig. (**5**) has a typical trend: it grows linearly, and then it rapidly falls down to zero. In other words, for either very little or very large values of the CUR, the effective transport capacity is very small. In order to understand this behavior, we first observe the following facts. For a given network area *A*, from (14) one can write:

$$C_{\mathrm{Te}} \propto \mathrm{CUR}\, R_{\mathrm{b}}\, \overline{n}_{\mathrm{sh}} \qquad (15)$$

in which $\overline{n}_{\mathrm{sh}}$ is a function of the link SNR in (2), where $P_{\mathrm{thermal}} \propto R_{\mathrm{b}}$ and P_{int} is an increasing function of the CUR (almost linear, as indicated in (12)). At this point, one can understand what happens for varying values of the CUR.

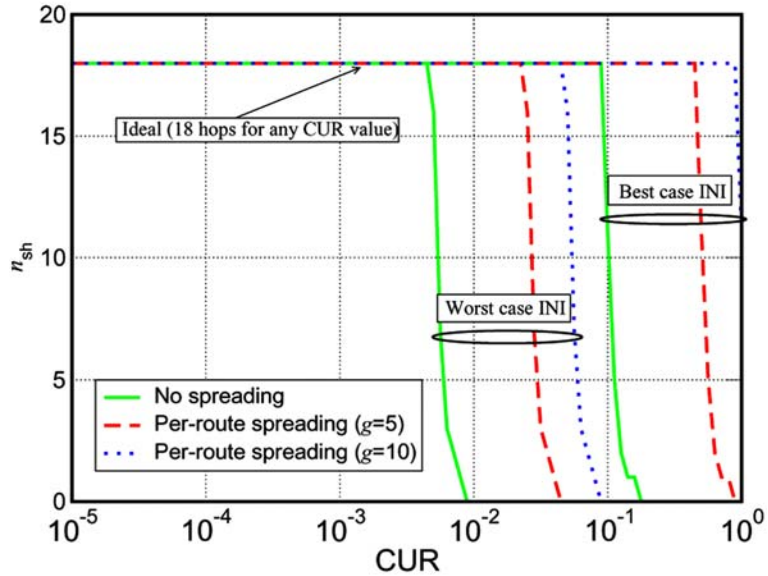

Fig. (6). Average sustainable number of hops as a function of the channel utilization ratio. A realistic case (with best-case and worst-case INI scenarios) and the ideal (no INI) case are considered.

- For small values of the CUR, the interference power is negligible, and the total noise power is given by the thermal noise power. In particular, C_{Te} grows almost linearly with the CUR. This means that R_b is almost constant and $P_{thermal}$ is sufficiently low to guarantee full connectivity, i.e.,

  $\overline{n}_{sh} = \overline{n}_h = \lfloor N/\pi \rceil = \lfloor 1000/\pi \rceil = 18.$

- The optimized value of the CUR corresponds to a scenario where thermal and interference noise powers are, together, still sufficiently small to guarantee full connectivity. However, for values of the CUR larger than the optimized one, the interference power becomes too large, and connectivity is lost. This is shown in Fig. (**6**), where the average number of sustainable hops $\overline{n}_{sh}$ is shown. In the ideal case, the average number of sustainable hops is equal to the average number $\overline{n}_h$ for all CUR values. Loss of connectivity, i.e., sudden drop of $\overline{n}_{sh}$, leads to rapid deterioration of the effective transport capacity to zero. This sudden loss of connectivity, for increasing values of the CUR, i.e., for increasing interference, is in agreement with the conclusions, based on percolation theory, presented in [13] for ad hoc wireless networks affected by interference.

Summarizing: for low values of CUR, the shared resource (radio channel) is underutilized, there is full connectivity and the effective transport capacity is low; for large values of CUR, the channel is overutilized, the interference level is too large, connectivity is lost, and the effective transport capacity drops to zero. There is a sharp transition between these two regions, and the optimal CUR is exactly in the middle.

7.2. Impact of Spreading

Observing carefully the results shown in Fig. (**5**), one can immediately realize that the improvement brought by using per-route spreading codes with g=10 is not twice as large as that brought by using per-route spreading codes with g=5. In other words, this means that it is sufficient to consider a relatively limited set of per-route spreading codes to significantly improve the network performance. Use of larger and larger sets of per-route spreading codes leads, in relative terms, to smaller and smaller performance improvement.

7.3. Impact of Transmit Power

The results shown in Figs. (**5**,**6**) are obtained considering a transmit power equal to P_t=1 mW (common for all nodes). This corresponds, for the considered node spatial density (ρ_S=10^{-3} m^{-2}), to requiring that the receiver sensitivity of a node is around -110 dBm. Our results on the impact of the transmit power on the network performance lead to the following conclusions.

- Increasing the transmit power (for example, considering P_t=10 mW and a receiver sensitivity of -100 dBm) leads to negligible performance improvement, in terms of effective transport capacity. In fact, the only effect is that of making loss of connectivity faster for values of CUR higher than the optimal value. This is due to the fact that in the region where interference power dominates and connectivity is lost, i.e., for values of CUR larger than the optimized value, the link SNR becomes insensitive to the transmit power (since both the received signal power P_t and the interference noise power P_{int} are proportional to the transmit power P_t, and the thermal noise power $P_{thermal}$ is negligible). In other words, this means that increasing the transmit power corresponds to a waste of resources, especially in terms of battery consumption at the nodes.

- On the other hand, if the transmit power is reduced, it turns out that the critical behavior around the optimized CUR value (i.e., full connectivity below it and loss of connectivity right above it) disappears. More precisely, the optimized CUR value is no longer a critical connectivity value; instead, there is also a region, above this optimized value, where connectivity is still preserved. In other words, connectivity does not break suddenly as soon as the CUR slightly overcomes the optimal CUR value. In this reduced transmit power scenario, the optimal CUR corresponds to a *stable* network behavior: small oscillations of the actual CUR around the optimal value do not lead to

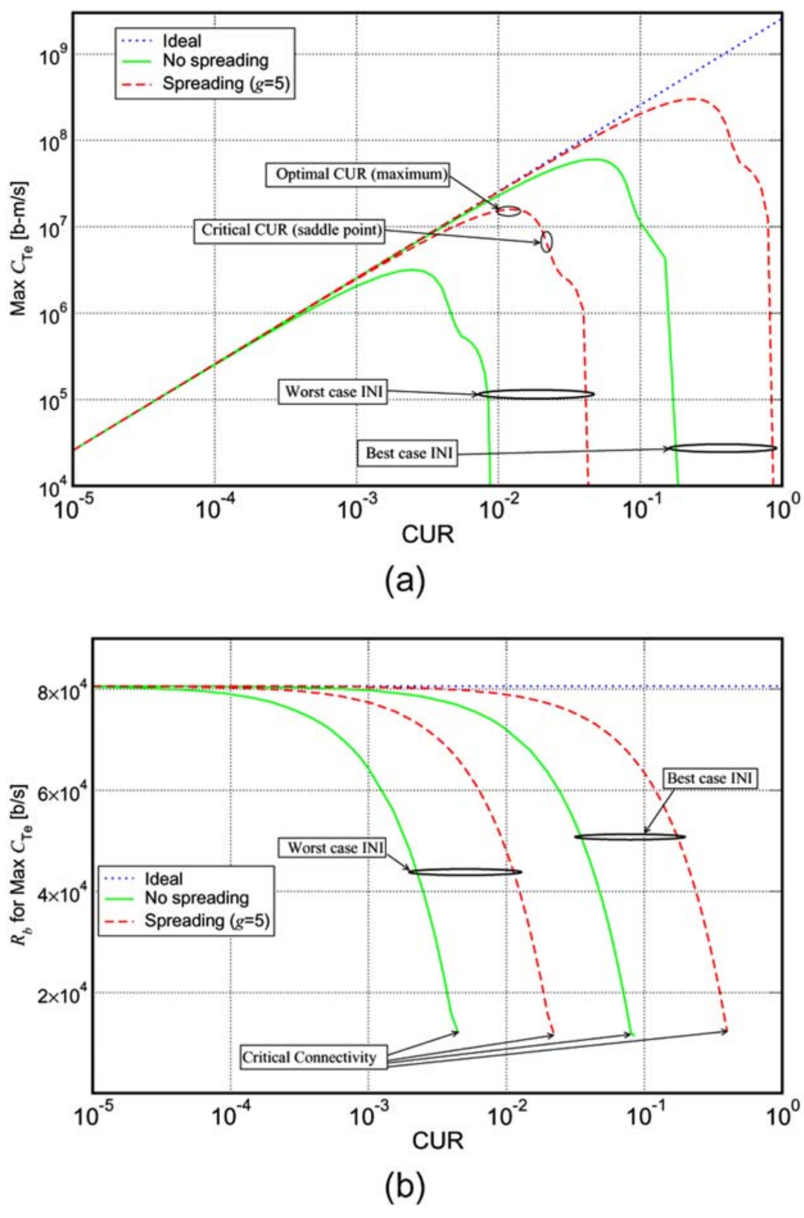

Fig. (7). Optimized network performance in an "idealized" scenario with P_t=0.1 μW: (**a**) effective transport capacity as a function of the CUR, and (**b**) optimized data-rate as a function of the CUR. Realistic (with best-case and worst-case INI scenarios) and ideal (no INI) cases are considered. In realistic scenarios, the presence and the absence of per-route spreading codes are considered.

loss of connectivity. Reducing the transmit power, however, requires a proportional increase of the receiver sensitivity, and this might be unrealistic for a practical ad hoc wireless network. For example, the performance of the effective transport capacity in an "idealized" scenario with P_t=0.1 μW is shown in Fig. (**7a**), whereas in Fig. (**7b**) the corresponding best data-rate is shown as a function of the CUR.

This scenario is idealized, since, for the considered network topology ($\rho_S=10^{-3}$ m^{-2}) the receiver sensitivity should be -150 dBm, which is obviously unthinkable (at least with the current technology for wireless sensor networks). However, it is expedient to show the positive impact, from a stability point of view, of a transmit power reduction.

The key observation in this case is that it is possible to trade receiver sensitivity for stability. In order to obtain a stable behavior with an acceptable receiver sensitivity, one possibility would be the use of directional antennas [18, 19]. This research direction is currently under investigation.

7.4. Absence of Synchronization and/or Fairness

We first discuss on the validity of the obtained results, in terms of the existence of an optimized CUR for maximization of the effective transport capacity, in a networking scenario where there is not perfect synchronization among the RCUIs of different active nodes. Since we have shown that the performance difference between networking scenarios with continuous and discontinuous transmission strategies is minimal, we refer to a network communication scenario with discontinuous transmission strategy. Assuming no synchronization among the nodes, *but* still assuming that when a route is torn down another one is immediately activated, the communication matrix model in Fig. (**4**) modifies to the model shown in Fig. (**4**).

In particular, in each row two consecutive messages are shown, each one constituted by k=13 bits spread randomly over n=27 positions. The dashed lines at the edges of the figure refer to other messages. In particular, in the center of the figure, a "box" relative to a "virtual" RCUI with n bit positions is indicated. Due to the randomized positions of the bits in the rows, it is possible to evaluate the interference power as previously done and, therefore, obtain the same optimized CUR value for the maximization of the effective transport capacity.

Suppose now that fairness is violated and, for example, a node reserves a route for $2n$ time units. For instance, this corresponds, in Fig. (**8**), to assuming that the consecutive messages indicated in each row are transmitted by the same node and not by different nodes. Obviously, the matrix model in Fig. (**8**) is still valid *provided that* the CUR is fixed. In other words, if the RCUI of a node is longer, then the corresponding message duration, i.e., the ECUI duration, is proportionally longer. Therefore, the results on the existence of an optimized CUR value would not change.

We note that the fact that the optimized CUR is insensitive to synchronization among the nodes and to the fairness policy enforced in the network, makes it a meaningful ad hoc wireless network performance indicator.

8. CONCLUDING REMARKS

The impact of the CUR on the performance of ad hoc wireless networks has been investigated. Each node, after reserving a multi-hop route to its destination, holds this route for an RCUI and effectively uses it for an ECUI. We have considered an ideal scenario (without INI) and a realistic scenario (with INI). In the latter case, the use of *per-route spreading codes* has been proposed as an effective means to reduce the interference. It has been shown that small or large values of the CUR correspond to a small effective transport capacity, because of radio resource underutilization or excessive interference, respectively. Our results have shown the existence of an *optimized* intermediate value of the CUR for

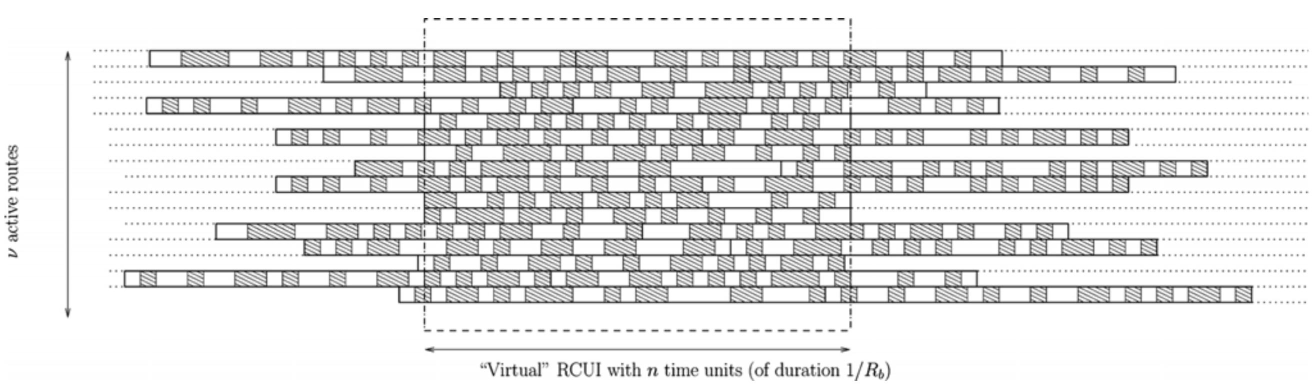

Fig. (8). Networking communication matrix model with unsynchronized active nodes. In each row, two *discontinuous* messages k time units long (over a RCUI n time units long) are "flowing". In the figure, ν=17, n=27, and k=13.

the maximization of the effective transport capacity. This CUR value corresponds to a scenario where the two effects mentioned above, i.e., resource underutilization and interference, are best balanced. Our results show that use of per-route spreading codes with relatively low spreading factor might lead to significant performance improvement. On the other hand, it is important to understand that increasing the transmit power beyond a critical point does not increase the effective transport capacity. It has also been shown that the particular message transmission strategy, either *continuous* or *discontinuous*, entails negligible differences in terms of performance and, consequently, on the value of the optimized CUR. This confirms that the CUR is a meaningful network performance indicator.

As mentioned in the introduction, the impact of the CUR on the network performance has been analyzed considering several simplifying assumptions. However, even relaxing these assumptions, it would still be possible to identify an optimized CUR in the corresponding wireless network communication scenario. Our current research is focused on exploring this interesting extension.

A. PROBABILITY OF BIT INTERFERENCE FROM A SINGLE ROUTE

In this appendix, we evaluate the probability of bit interference from a single route. In particular, we first evaluate the probability of interference as a function of the bit position inside the RCUI, and then we consider an average value, independent of the specific bit position.

A.1. Continuous Transmission

In this case, the k bits of a message generated by a source node are transmitted continuously, after randomly choosing the initial instant for message transmission within the RCUI. This scenario is shown in Fig. (**3**). Since there are n possible positions in an RCUI, it follows that there are (n-k+1) possible positions for the first bit of the message to be transmitted. In order to analyze the probability of bit interference from a single route, it is expedient to distinguish between the two situations with (i) $n \geq 2k$ and (ii) $k \leq n < 2k$.

A.1.1. Case with n≥2k

It is easy to distinguish among the bit positions as follows.

- i:1→k: there is interference in the bit position i if the message transmission in the other route starts in any bit position between the first and the i-th. Hence, there are i starting positions which cause interference. Assuming that the choice of the starting position of a transmission is a random value between 1 and (n-k+1), it follows that

$$\mathrm{p}_{\mathrm{int,c}}^{(i)} = \frac{i}{n-k+1}.$$

- i:k+1→n-k+1: there is interference if the message transmission in another route starts in a bit position between the (i-k+1)-th and the i-th. Hence, there are k possible starting positions causing interference, so that

$$\mathrm{p}_{\mathrm{int,c}}^{(i)} = \frac{k}{n-k+1}.$$

- i:n-k+2→n: recalling that the last possible position for the first bit of a message is the (n-k+1)-th, there is interference if the packet transmission in the other route starts between the (i-k+1)-th position and the (n-k+1)-th position. Hence, there are (n-i+1) positions generating interference, and one can conclude that

$$\mathrm{p}_{\mathrm{int,c}}^{(i)} = \frac{n-i+1}{n-k+1}.$$

Summarizing, the probability of bit interference can be written, as a function of the bit position, as follows:

$$\mathrm{p}_{\mathrm{int,c}}^{(i)} = \begin{cases} \dfrac{i}{n-k+1} & 1 \leq i \leq k \\ \dfrac{k}{n-k+1} & k+1 \leq i \leq n-k+1 \\ \dfrac{n-i+1}{n-k+1} & n-k+2 \leq i \leq n. \end{cases}$$

Note that $\mathrm{p}_{\mathrm{int,c}}^{(i)} = \mathrm{p}_{\mathrm{int,c}}^{(n-i+1)}, i \in \{1, \ldots, \lfloor n/2 \rfloor\}$, i.e., the distribution of the probability of bit interference is symmetric with respect to the center of the possible bit positions. In order to determine an average probability of bit interference from a single route, one needs to compute the probability of bit transmission in the position i. We indicate this probability distribution with the notation[8] $\{\mathrm{p}_{\mathrm{tx}}(i)\}$. In order to determine this probability distribution, for each position i we evaluate the number of transmission configurations such that *there is* a bit in position i. The following cases can be distinguished.

- i:1→k-1: there is a bit in position i if the packet transmission starts in any bit position between the first and the i-th. Hence, there are i possible starting positions.
- i:k→n-k+1: there is a bit in position i if the packet transmission starts in any bit position between the (i-k+1)-th and the i-th. Hence, there are k possible positions.
- i:n-k+2→n: recalling that the last starting position for a packet transmission is the (n-k+1)-th, we can conclude that there is a bit in this position if the packet transmission starts in any position between the (i-k+1)-th and the (n-k+1)-th. Hence, there are (n-i+1) possible positions.

From the computation above it is possible to derive a probability distribution by normalizing the numbers of possible starting positions (for i=1,...,n) with their sum, i.e., with $2\sum_{i=1}^{k} i + (n-k+2)k = k(n-k+1)$, obtaining:

[8] Unlike for the distribution $\{\mathrm{p}_{\mathrm{int,c}}^{(i)}\}$, for the distribution $\{\mathrm{p}_{\mathrm{tx,c}}(i)\}$ the condition $\sum_{i=1}^{n} \mathrm{p}_{\mathrm{tx,c}}(i) = 1$ needs to be respected.

$$\mathrm{p_{tx,c}}(i) = \begin{cases} \dfrac{i}{k(n-k+1)} & i = 1, \ldots, k-1 \\ \dfrac{1}{n-k+1} & i = k, \ldots, n-k+1 \\ \dfrac{n-i+1}{k(n-k+1)} & i = n-k+2, \ldots, n. \end{cases}$$

At this point, the average probability of bit interference, indicated as , can be computed as follows:

$$\begin{aligned} \overline{\mathrm{p}}_{\mathrm{int,c}} &= \sum_{i=1}^{n} \mathrm{p}_{\mathrm{int,c}}^{(i)} \, \mathrm{p_{tx,c}}(i) \\ &= \frac{-4k^2 + 3nk + 3k + 1}{3(n-k+1)^2} \qquad n \geq 2k. \end{aligned}$$

A.1.2. Case with k≤n<2k

In this case, regardless of the starting position of a message transmission, the positions between the (n-k+1)-th and the k-th are always occupied by bits. Reasoning as in the case with $n \geq 2k$, it is easy to distinguish the following cases, relative to position i, for the probability of bit interference.

- i:1→n-k: in this case there is interference if the message in the other route starts in any of the positions between the first and the i-th. Hence, there are i possibilities, and the probability of interference is

$$\mathrm{p}_{\mathrm{int,c}}^{(i)} = \frac{i}{n-k+1}.$$

- i:n-k+1→k: this bit position is always "covered" by another route, so that

$$\mathrm{p}_{\mathrm{int,c}}^{(i)} = 1.$$

- i:k+1→n: in this case, there is interference if the message transmission in another route starts in any position between the (i-k+1)-th and the (n-k+1)-th. Hence, there are n-i+1 positions, so that

$$\mathrm{p}_{\mathrm{int,c}}^{(i)} = \frac{n-i+1}{n-k+1}.$$

Summarizing, the probability of bit interference can be written, as a function of the bit position, as follows:

$$\mathrm{p}_{\mathrm{int,c}}^{(i)} = \begin{cases} \dfrac{i}{n-k+1} & 1 \leq i \leq n-k \\ 1 & n-k+1 \leq i \leq k \\ \dfrac{n-i+1}{n-k+1} & k+1 \leq i \leq n. \end{cases}$$

As considered in the case with $n \geq 2k$, in the case with $k \leq n < 2k$ as well, in order to compute an average probability of bit interference, we need to determine the distribution of the probability of bit transmission as a function of the position i. As in the case with $n \geq 2k$, we first count the number of possible transmission configurations as a function of the bit position, and then we normalize with respect to their sum.

- i:1→n-k: in order to have a bit transmission in position i, a message transmission must start in a position between 1 and i. In other words, there are i possible message starting positions.
- i:n-k+1→k: this position is always covered, regardless of the message position within the RCUI, i.e., all the possible (n-k+1) starting positions necessarily involve a bit transmission in position i.
- i:k+1→n: in order to have a bit transmission in position i, a message transmission must start in a position between the (i-k+1)-th position and the (n-k+1)-th position. Hence, there are (n-i+1) possible starting positions.

At this point, it is immediate to determine the probability of "existence" of a bit in position i by normalizing the number of interfering transmission cases with their sum (given by $2\sum_{i=1}^{n-k} i + (2k-n)(n-k+1) = k(n-k+1)$).
The following distribution is obtained:

$$\mathrm{p_{tx,c}}(i) = \begin{cases} \dfrac{i}{k(n-k+1)} & i = 1, \ldots, n-k \\ \dfrac{1}{k} & i = n-k+1, \ldots, k \\ \dfrac{n-i+1}{k(n-k+1)} & i = k+1, \ldots, n. \end{cases}$$

It is then possible to compute an average probability of interference by weighing the probability of bit interference $\mathrm{p}_{\mathrm{int,c}}^{(i)}$ with the probability distribution of bit existence $\mathrm{p_{tx}}(i)$, obtaining:

$$\begin{aligned} \overline{\mathrm{p}}_{\mathrm{int,c}} &= \sum_{i=1}^{n} \mathrm{p}_{\mathrm{int,c}}^{(i)} \, \mathrm{p_{tx,c}}(i) \\ &= \frac{-n^2 - 4k^2 + 5nk - 2n + 5k}{3k(n-k+1)} \\ & \qquad k \leq n < 2k. \end{aligned}$$

A.2. Discontinuous Transmission

In this case, a source node transmits the stored message "randomly" across the RCUI. This transmission scenario is depicted in Fig. (**4**). In each route there are $\binom{n}{k}$ possible combinations for the k bits over the n possible positions. A specific bit position will be interfered by a transmission in another route if, in the latter route, there is a bit in the same position. Hence, the number of combinations generating interference is given by the number of combinations of the remaining (k-1) bits (excluding the one fixed in the position of interest) in the remaining (n-1) positions, i.e., $\binom{n-1}{k-1}$. It is thus immediate to conclude that the probability of interference in a specific bit position from another single route does not depend on the considered position. Indicating this probability as $\mathrm{p_{int,d}}$, it can be written as follows:

$$\mathrm{p_{int,d}} = \frac{\binom{n-1}{k-1}}{\binom{n}{k}} = \frac{k}{n}.$$

This expression for the probability of interference is valid both for the case with $n \geq 2k$ and for the case with $k \leq n < 2k$. In order to compute an average probability of interference, the distribution of the probability of "existence" of a bit in position i (in other words, of a bit transmission in position i) needs to be computed. For any bit position, there are $\binom{n-1}{k-1}$ possible transmission configurations which cover the i-th bit position. Hence, we can conclude that the probability of existence of a bit, indicated as $\mathrm{p}_{\mathrm{tx,d}}(i)$, is uniformly distributed across the bit positions:

$$\mathrm{p}_{\mathrm{tx,d}}(i) = \frac{\binom{n-1}{k-1}}{n\binom{n-1}{k-1}} = \frac{1}{n} \qquad i = 1, \ldots, n.$$

Finally, it follows that the average probability of bit interference in the case of discontinuous transmission is

$$\overline{\mathrm{p}}_{\mathrm{int,d}} = \sum_{i=1}^{n} \mathrm{p}_{\mathrm{int,d}}\mathrm{p}_{\mathrm{tx,d}}(i) = \frac{k}{n}.$$

A.3. Summary of Results

The results obtained in Section A.1 and Section A.2 for the average probability of interference in a bit position from another single route are summarized in Table **3**.

Table 3. Summary of the Probability of Bit Interference from a Single Route

	$k \le n < 2k$	$n \ge 2k$
$\mathrm{p}_{\mathrm{int,c}}$	$\frac{-n^2-4k^2+5nk-2n+5k}{3k(n-k+1)}$	$\frac{-4k^2+3nk+3k+1}{3(n-k+1)^2}$
$\mathrm{p}_{\mathrm{int,d}}$	$\frac{k}{n}$	$\frac{k}{n}$

In Fig. (**9**), we compare the average probability of bit interference (as a function of n) in the case of continuous ($\overline{\mathrm{p}}_{\mathrm{int,c}}$) and discontinuous ($\overline{\mathrm{p}}_{\mathrm{int,d}}$) transmission, for various values of k.

In all cases, we consider $n \in \{k, k+1, \ldots, 10k\}$. It is immediate to notice that

$$\overline{\mathrm{p}}_{\mathrm{int,c}} > \overline{\mathrm{p}}_{\mathrm{int,d}} \quad \forall n.$$

It is possible to show that, for any value of k, the largest difference between $\overline{\mathrm{p}}_{\mathrm{int,c}}$ and $\overline{\mathrm{p}}_{\mathrm{int,d}}$ is obtained for a value of n slightly lower than $2k$, so that

$$\begin{aligned}\max_{n}(\overline{\mathrm{p}}_{\mathrm{int,c}} - \overline{\mathrm{p}}_{\mathrm{int,d}}) &\simeq (\overline{\mathrm{p}}_{\mathrm{int,c}} - \overline{\mathrm{p}}_{\mathrm{int,d}})_{|n=2k} \\ &= \frac{1}{6}\frac{k-1}{k+1}.\end{aligned}$$

Hence, we can conclude that the maximum value of the difference $(\overline{\mathrm{p}}_{\mathrm{int,c}} - \overline{\mathrm{p}}_{\mathrm{int,d}})$ is around 1/6 (for sufficiently large values of k). It is also possible to show that $\overline{\mathrm{p}}_{\mathrm{int,c}} - \overline{\mathrm{p}}_{\mathrm{int,d}} \simeq 2 \times 10^{-2}$ for $n \simeq 6k$. Finally, the difference between the two probabilities vanishes for increasing value of n: in other words, for $n \gg k$ (or, equivalently, CUR<<1) the probability of interference with continuous or discontinuous transmission is basically the same.

B. SINGLE ROUTE AVERAGE INTERFERENCE POWER

In this appendix, we evaluate the average interference power, experienced at the receiving node of a link, due to another single active route. In particular, we derive *upper* and *lower* bounds on the interference power generated by a route, which lead to lower and upper bounds on the effective

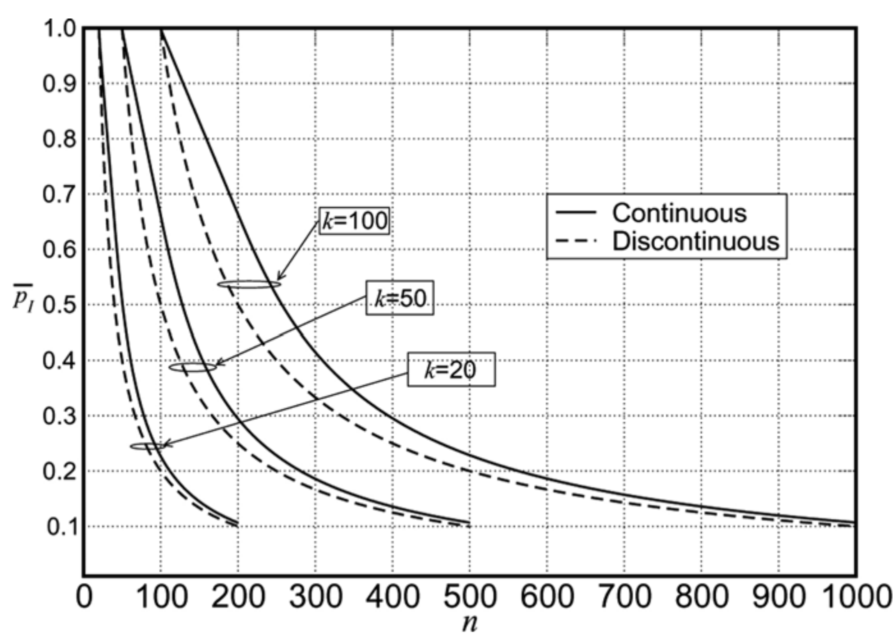

Fig. (9). Comparison between the average probability of bit interference from a single route in the case of continuous ($\overline{\mathrm{p}}_{\mathrm{int,c}}$) and discontinuous ($\overline{\mathrm{p}}_{\mathrm{int,d}}$) transmission as a function of n, for various values of k.

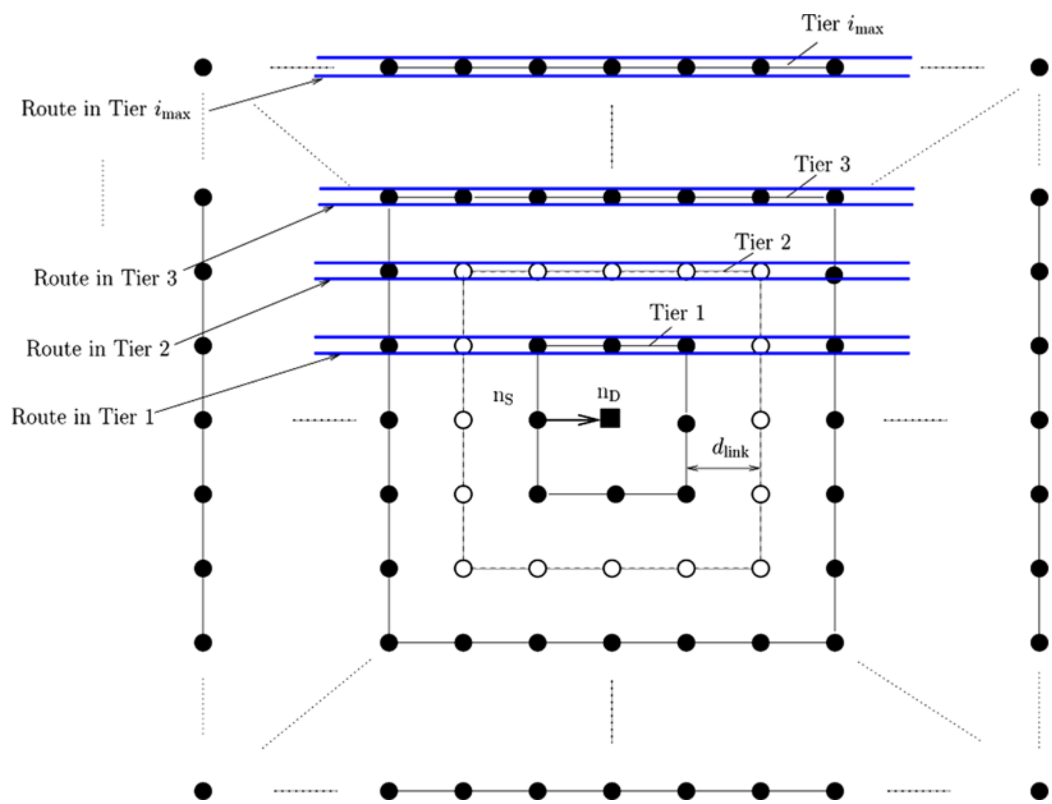

Fig. (10). Possible positions, from the closest (Tier 1) to the farthest (Tier i_{max}), for routes interfering at the central receiving node.

transport capacity, respectively. According to the matrix-based network communication model considered in this paper for the evaluation of the INI, there is interference between two routes in a specific bit position if that position is filled in both rows (corresponding to the two routes) of the matrix. At this point, provided that there is superposition, the actual amount of the interference power at the end of a link of the multi-hop route of interest needs to be computed, and it obviously depends on the spatial distribution and activity of the nodes in each route. A simple way to determine upper and lower bounds on the interference power consists thus in assuming that only a node or all nodes of the interfering routes are active, respectively.

According to the topology assumption considered in this paper, we refer to a node spatial distribution where the nodes are at the vertices of a square grid. Representing the node distribution as a tiered structure, we assume that the central node n_D of the structure represents the node (receiving the message sent by a neighboring source node n_S) in correspondence to which we want to evaluate the bit interference power.[9] We assume, for analytical tractability, that the possible interfering routes are straight. In particular, they may be placed as indicated in Fig. (**10**).

More precisely, we characterize an interfering route according to the lowest-order tier over which the multi-hop route passes. In Fig. (**10**), it is shown that the closest route is the one containing a portion of Tier 1 (indicated in the figure as "Route in Tier 1"), whereas the farthest route is that leaning over Tier $i_{max} = \lfloor \sqrt{N}/2 \rceil$ (indicated as "Route in Tier i_{max}"). Note that the interfering routes might not be parallel to the considered link. However, the ensemble of routes shown in Fig. (**10**) allows one to determine a reasonable estimate for the interference power.

At this point, in order to evaluate the interference power generated from the route in Tier i, $i = 1, \ldots, i_{max}$, we assume that the route is symmetric with respect to the central position. As previously shown, a multi-hop communication route is formed, on average, by $\overline{n}_h = \lfloor \sqrt{N/\pi} \rceil$ successive hops. Hence, in an average route there are $\overline{n}_h + 1$ nodes.

In Fig. (**11**), the distances from the receiving node (and corresponding interference powers) of half of the interfering nodes are shown (due to symmetry, the distances, from n_D, of the other half of nodes are specular[10]). Simple geometric

[9]The position of the link $n_S \rightarrow n_D$ represents the worst possible position in the network from the point of view of the interference power.

[10]Note that in Fig. (11), we are assuming that $\overline{n}_h$ is an odd integer. However, the obtained results are, obviously, qualitatively the same also for even values of $\overline{n}_h$.

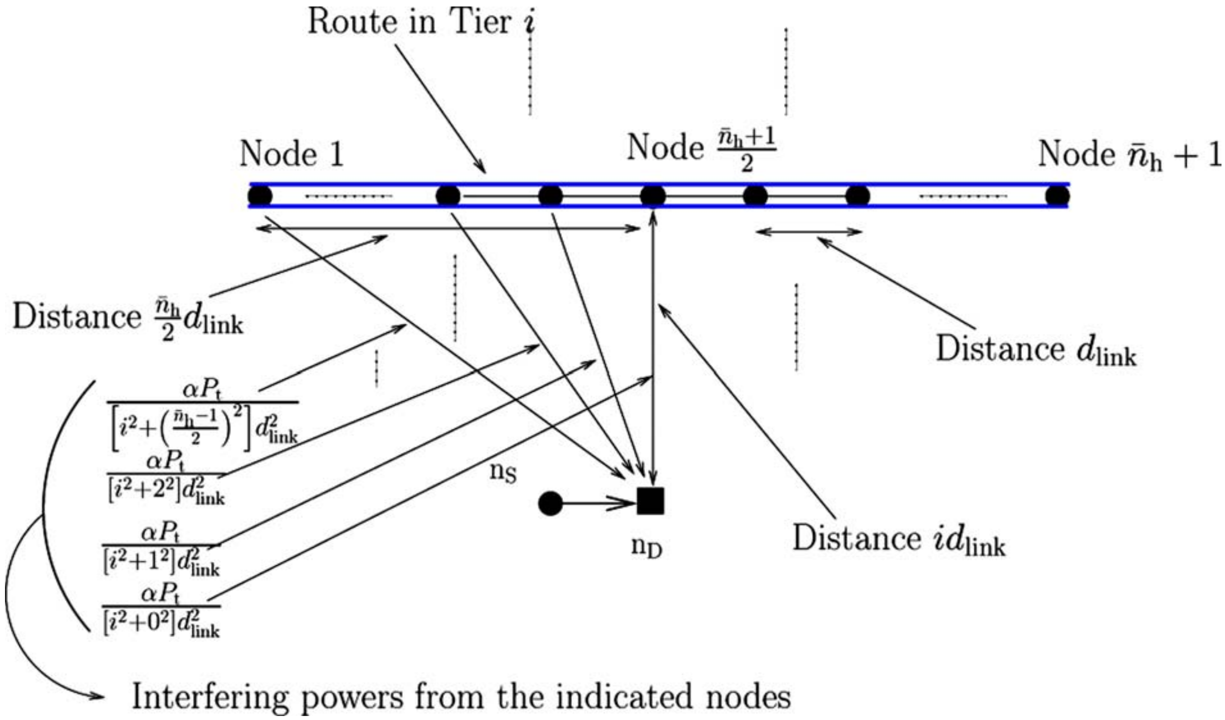

Fig. (11). Interference powers from the nodes belonging to the route in Tier i. The situation in the figure corresponds to an odd value of $\overline{n}_h$.

considerations show that the distance from n_D of a node in Tier i, the distance of which from the center of the route is jd_{link}, is $\sqrt{i^2+j^2}d_{\text{link}}$. Hence, the interference power originating from this node is $\alpha P_t/(i^2+j^2)d^2_{\text{link}}$. At this point, we distinguish between two possibilities regarding the interference power originating from a route in tier i.

- In a *worst-case* scenario, all the nodes of the interfering route are simultaneously active, so that the interference power is obtained by summing the interference powers generated at all nodes of the interfering route. Indicating by $\overline{P}^{(i)-\max}_{\text{int}}$ the (maximum) interference power generated by a route in the i-th tier, it follows:

$$\overline{P}^{(i)-\max}_{\text{int}} = \frac{P_t\alpha}{i^2 d^2_{\text{link}}} + 2\sum_{j=1}^{\lfloor\frac{\overline{n}_h}{2}\rceil} \frac{P_t\alpha}{(i^2+j^2)d^2_{\text{link}}} \quad i=1,\ldots,i_{\max}.$$

- In a *best-case* scenario, only a single node in the route is active during the transmission in the considered link of the reference route. In this case, indicating by $\overline{P}^{(i)-\min}_{\text{int}}$ the interference power generated by a route in the i-th tier, it follows:

$$\overline{P}^{(i)-\min}_{\text{int}} = \frac{\overline{P}^{(i)-\max}_{\text{int}}}{\overline{n}_h+1} = \frac{\frac{P_t\alpha}{i^2 d^2_{\text{link}}} + 2\sum_{j=1}^{\lfloor\frac{\overline{n}_h}{2}\rceil}\frac{P_t\alpha}{(i^2+j^2)d^2_{\text{link}}}}{\overline{n}_h+1} \quad i=1,\ldots,i_{\max}.$$

In a generic situation, a number of nodes between 1 and $\overline{n}_h+1$ could be active in the interfering route, so that we can assume that in general the interference power will be between $\overline{P}^{(i)-\min}_{\text{int}}$ and $\overline{P}^{(i)-\max}_{\text{int}}$. Finally, we assume that the average interference power due to a single route can be written as the arithmetic average of the interference powers due to the routes in the successive tiers. This average interference power from a single route, indicated as $\overline{P}^{(i)-\min}_{\text{int}}$ and $\overline{P}^{(i)-\max}_{\text{int}}$ in the worst-case and best-case scenarios, respectively, can be written as follows:

$$\overline{P}^{(\text{sr})-\max}_{\text{int}} = \frac{1}{i_{\max}}\sum_{i=1}^{i_{\max}}\overline{P}^{(i)-\max}_{\text{int}} = \frac{\alpha P_t \rho_S}{i_{\max}}\sum_{i=1}^{i_{\max}}\left(\frac{1}{i^2} + 2\sum_{j=1}^{\lfloor\frac{\overline{n}_h}{2}\rceil}\frac{1}{i^2+j^2}\right)$$

$$\overline{P}^{(\text{sr})-\min}_{\text{int}} = \frac{1}{i_{\max}}\sum_{i=1}^{i_{\max}}\overline{P}^{(i)-\min}_{\text{int}} = \frac{\alpha P_t \rho_S}{i_{\max}(\overline{n}_h+1)}\sum_{i=1}^{i_{\max}}\left(\frac{1}{i^2} + 2\sum_{j=1}^{\lfloor\frac{\overline{n}_h}{2}\rceil}\frac{1}{i^2+j^2}\right)$$

which corresponds to (9) and (10), respectively.

REFERENCES

[1] P. Gupta and P. R. Kumar. The capacity of wireless networks. *IEEE Trans. Inform. Theory*, 46(2):388–404, March 2000.

[2] C. E. Perkins. *Ad hoc Networking*. Addison-Wesley, Upper Saddle River, NJ, USA, 2001.

[3] Y. Y. Kim and S. Li. Modeling multipath fading channel dynamics for packet data performance analysis. *Wireless Networks*, 6:481–492, 2000.

[4] M. Zorzi and S. Pupolin. Optimum transmission ranges in multihop packet radio networks in the presence of fading. *IEEE Trans. Commun.*, 43:2201–2205, July 1995.

[5] M. Haenggi. On routing in random Rayleigh fading networks. *IEEE Trans. Wireless Commun.*, 4(4):1553–1562, July 2005.

[6] O. K. Tonguz and G. Ferrari. *Ad Hoc Wireless Networks: A Communication-Theoretic Perspective*. John Wiley & Sons, 2006.

[7] T. J. Shepard. A channel access scheme for large dense packet radio networks. In *Proc. ACM Conference of the Special Interest Group on Data Communication* (SIGCOMM), pages 219–230, Palo Alto, CA, 1996.

[8] R. Rozovsky and P. R. Kumar. SEEDEX: a MAC protocol for ad hoc networks. In *Proc. ACM Int. Symp. On Mobile Ad Hoc Network and Comput.* (MOBIHOC), pages 67–75, Long Beach, CA, USA, 2001.

[9] S. Panichpapiboon, G. Ferrari, N. Wisitpongphan, and O. K. Tonguz. Route reservation in ad hoc networks. *IEEE Trans. Mobile Comput.*, 6(1):56–71, January 2007.

[10] S. Panichpapiboon, G. Ferrari, and O. K. Tonguz. Optimal transmit power in wireless sensor networks. *IEEE Trans. Mobile Comput.*, 5(10), October 2006.

[11] G. Ferrari and O. K. Tonguz. Impact of mobility on the BER performance of ad hoc wireless networks. *IEEE Trans. Veh. Technol.*, 56(1):271–286, January 2007.

[12] Insitute of Electrical and Electronics Engineers. IEEE Std 802.11b-1999, 1999.

[13] M. Franceschetti, O. Dousse, D. Tse, and P. Thiran. Closing the gap in the capacity of wireless networks *via* percolation theory. *IEEE Trans. Inform. Theory*, 53(3):1009–1018, March 2007.

[14] T. H. Cormen, C. E. Leiserson, R. L. Rivest, and C. Stein. *Introduction to Algorithms*. MIT Press, Cambridge, Mass., second edition, 2002.

[15] G. Ferrari, B. Baruffini, and O. K. Tonguz. Spectral efficiency-connectivity tradeoff in ad hoc wireless networks. In *Proc. IEEE Symposium on Information Theory and Applications* (ISITA), pages 451–456, Parma, Italy, October 2004.

[16] T. S. Rappaport. *Wireless Communications. Principles & Practice,* 2nd Edition. Prentice-Hall, Upper Saddle River, NJ, USA, 2002.

[17] N. Abramson. Multiple access in wireless digital networks. *Proc. IEEE*, 82(9):1360–1370, September 1994.

[18] S. Yi, Y. Pei, and S. Kalyanaraman. On the capacity improvement of ad hoc wireless networks using directional antennas. In *Proc. ACM Int. Symp. on Mobile Ad Hoc Network. and Comput.* (MOBIHOC), pages 108–116, Annapolis, MA, USA, June 2003.

[19] R. Ramanathan, J. Redi, C. Santivanez, D. Wiggins, and S. Polit. Ad hoc networking with directional antennas: a complete system solution. *IEEE J. Select. Areas Commun.*, 23(3):496–506, March 2005.

Received: July 5, 2007 Revised: August 7, 2007 Accepted: October 7, 2007