

## Full length article

# Impact of the knowledge of nodes' positions on spectrum sensing strategies in cognitive networks



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## ABSTRACT

In this paper, we focus on cognitive wireless networking, where a primary wireless network (PWN) is co-located with a cognitive (or secondary) wireless network (CWN). The shared frequency spectrum is divided into disjoint “subchannels” and each subchannel is “freely” assigned (in a unique way) to a node of the PWN, denoted as primary user equipment (PUE). We assume that the nodes of the CWN, denoted as cognitive user equipments (CUEs), cooperate to sense the frequency spectrum and estimate the idle subchannels which can be used by the CWN (i.e., assigned to CUEs) without interfering the PWN. The sensing correlation among the CUEs is exploited to improve the reliability of the decision, taken by a secondary fusion center (FC), on the occupation status (by a node of the PWN) of each subchannel. In this context, we compute the mutual information between the occupation status and the observations at the FC, with and without knowledge of the positions of the nodes in the network, showing a potential significant benefit brought by this side information. Then, we derive the fusion rules at the FC: our numerical results, in terms of the network-wise probabilities of missed detection (MD) and false alarm (FA) at the secondary FC, indicate a significant performance improvement when knowledge of the CUEs' positions is available at the secondary FC, confirming the mutual information-based theoretical prediction.

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## 1. Introduction

Dynamic spectrum access has been proposed to provide efficient radio spectrum utilization [1–3]. In such systems, a portion of the spectrum can be allocated to one or more users, which are called primary user equipments (PUEs). Such spectrum, however, may not be exclusively

dedicated to PUEs, but could also be utilized, with lower priority, by secondary users, also denoted as cognitive user equipments (CUEs)—the notation comes from cellular systems where the proposed techniques can also be applied. In particular, CUEs can access the same spectrum (as long as the PUEs are not using it at that moment) or can share the spectrum with the PUEs (as long as the PUEs can be properly protected from undesired interference). By doing so, the radio spectrum can be reused in an opportunistic manner or shared all the time, thus significantly improving the spectrum utilization efficiency.

To support dynamic spectrum access, CUEs are required to sense the radio environment, i.e., they also are cognitive radio users [4,5]. One of the main tasks of a CUE is represented by spectrum sensing, defined as the task of

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finding portions of spectrum licensed to some PUEs but left unused for a certain amount of time [6]. Sensing from a single node does not always guarantee satisfactory performance because of the following: sensing noise; the intrinsic random nature of the nodes' positions; and unpredictable channel fluctuations. For example, a CUE could not detect the signal from a primary transmitter behind a high building and could decide to access a licensed subchannel, thus partially interfering with the primary receiver. On the other hand, collaboration of multiple users may highly improve spectrum sensing performance by introducing a form of spatial diversity [7,8]. In cooperative spectrum sensing, CUEs send the collected data to a combining user or fusion center (FC). Alternatively, CUE may first independently decide on the statuses of the subchannels and report binary decisions to the FC, which uses such data to take a decision on the occupation of each subchannel.

Although a well-established technique, great attention has recently been paid to cognitive radio since it has been identified as a key enabling technology for next-generation 5G systems [9]. Among all possible usages of cognitive radio in 5G scenario, a very interesting application lies in the field of the so-called green communications [10], i.e., the design of wireless infrastructures with limited cost and energy consumption. As an example, in [11] the authors propose green cognitive relaying, where data transmissions opportunistically occur when spectrum holes are identified, whereas energy harvesting is performed when PUEs occupy the licensed spectrum.

In this paper, we focus on cognitive wireless networking, where a primary (i.e., licensed) wireless network (PWN) is co-located with a cognitive (or secondary) wireless network (CWN). In particular, the nodes of the CWN reach their associated access point (AP) directly (i.e., single hop communications are assumed). The nodes of the CWN cooperate to sense the frequency spectrum and estimate the subchannels unused by the nodes of the PWN. The CUEs transmit packets containing the observations on the channels' statuses (idle or busy) to their FC, "embedded" in the secondary AP, which makes a final decision about the status of each subchannel and broadcasts this information to all CUEs. In this context, we first derive an expression for the mutual information between the occupation status and the observations at the FC. Then, optimal fusion rules, with and without the knowledge of the positions of the nodes, are derived and the missed detection (MD) and false alarm (FA) probabilities are computed to obtain system receiver operating characteristic (ROC) curves [12]. Both approaches indicate a significant performance improvement when knowledge of the nodes' positions is available at the secondary FC. Our work is inspired by recent advances in wireless communications, where proper transmission and signal processing-aided schemes are designed to exploit the knowledge of nodes' positions [13,14].

This scenario has been preliminarily analyzed in [15], where the case without knowledge of the positions of the nodes in the network is considered. Note that related work is carried out in [16], where a scenario with CUEs with known positions, close to each other and far from a PUE, is considered. In this case, the sensing channels

are correlated and, therefore, sub-optimal fusion rules are devised to take into account this correlation. Unlike [16], here we consider a more realistic sensing scenario where CUEs are not necessarily close to each other and, therefore, channel impairments may be independent. However, the correlation between the decisions of the CUEs can be exploited if the secondary AP knows their positions, thus improving the network performance, in terms of MD and FA probabilities on the status of each subchannel.

The rest of this paper is structured as follows. In Section 2, we present the system model. In Section 3, we analyze the MD and FA probabilities from a single CUE perspective. In Section 4, we derive an information-theoretic framework to compute the ultimate performance limits, in terms of mutual information between the observation vector at the FC and the binary data representing the occupation status of a subchannel, of the considered cognitive networking scenario, distinguishing between the cases with and without knowledge of the positions of the nodes. Then, in Section 5 we derive optimal fusion rules at the FC, with and without knowledge of nodes' positions, evaluating the MD and FA probabilities of the decision by a CUE on the occupation status of a subchannel. Finally, concluding remarks are given in Section 6.

## 2. System model

The scenario of interest is shown in Fig. 1. The FC, "embedded" in the secondary AP, is placed at the center of the region of interest (ROI), which is a circular cell with a given radius  $R$ , while CUEs and PUEs are independent and identically distributed (i.i.d.) according to a uniform distribution in the ROI.<sup>1</sup> The numbers of PUEs and CUEs are indicated as  $P$  and  $N$ , respectively. The PWN can operate on  $N_{\text{ch}}$  orthogonal subchannels corresponding to non-overlapping frequency bands, i.e., each PUE can transmit data on one of such  $N_{\text{ch}}$  channels. Each PUE is assigned one of the  $N_{\text{ch}}$  orthogonal subchannels to transmit its own data (when available) with fixed power  $P_T$ . Due to the assumption of orthogonal subchannels, in the rest of the paper we will focus, without loss of generality, on a generic subchannel. The binary status of the reference subchannel  $S$  is defined as follows:

$$S = \begin{cases} S_0 & \text{with probability } P(S_0) \\ S_1 & \text{with probability } P(S_1) = 1 - P(S_0). \end{cases}$$

Data transmissions follow a classical model for cellular environments, where the path-loss is completely characterized by two parameters: (i) the distance attenuation factor  $\alpha$  (adimensional, in the range  $2 \div 4$ ) and (ii) the standard deviation  $\sigma$  (in dB) of the log-normal shadowing [17].

Each CUE scans the subchannel in order to detect the presence of a primary signal transmission. In other words, each CUE performs a binary hypothesis test on the presence of a primary signal in the subchannel, which is idle under hypothesis  $S_0$  and busy under hypothesis  $S_1$ . The sensing time of the CUEs depends on the particular

<sup>1</sup> No assumption is done on the position of the primary AP, which, for instance, may be co-located with the secondary AP.

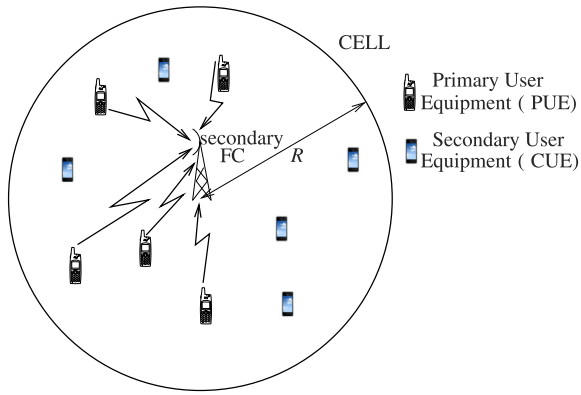


Fig. 1. System model for cognitive networking scenario of interest.

technology and has impact on the achievable throughput at the secondary FC. However, this goes beyond the scope of this paper and the interested reader may refer to the existing literature, e.g., [18]. As for the signal model, we assume that the primary signal can be modeled as a zero-mean stationary white Gaussian process—this is a reasonable assumption when the CWN has no a-priori knowledge about the possible modulation and pulse shaping formats adopted by the PWN.

On the basis of the model assumptions above, the  $k$ th CUE ( $k = 1, \dots, N$ ) has to distinguish between two independent Gaussian sequences:

$$\mu_k(\ell) = \begin{cases} n_k(\ell) & \text{if } S_0 \\ s_k(\ell) + n_k(\ell) & \text{if } S_1 \end{cases} \quad \ell = 1, \dots, m \quad (1)$$

where  $m$  is the number of observed consecutive samples in a single block used to generate a binary decision. Note that an implicit assumption in (1) is that the phenomenon status ( $S_0$  or  $S_1$ ) does not change over  $m$  consecutive observations: this is realistic given that  $m$  is typically much shorter than the duration of a primary signal transmission. Moreover, the fact that the  $k$ th node takes  $m$  consecutive observations  $\{\mu_k(\ell)\}_{\ell=1}^m$  on the subchannel guarantees time diversity in the sensing operation. In (1),  $s_k(\ell)$  is the signal received by the  $k$ th CUE, which is a sequence of i.i.d. zero-mean complex Gaussian random variables with variance  $P_R^{(k)}$  (corresponding to the received power). The power  $P_R^{(k)}$  depends on the transmitted power  $P_T$  and on the path-loss and shadowing terms of each CUE–PUE pair. In the following, we assume that the path-loss and shadowing terms are constant over all  $m$  acquisitions: this is reasonable in a sufficiently slowly varying wireless scenario. More precisely, typical values for the acquisition time between consecutive samples are on the order of a few milliseconds [19] and, therefore, the assumption is not critical for  $m$  on the order of a few tens. The noise terms  $\{n_k(\ell)\}$  are also modeled as i.i.d. zero-mean complex Gaussian random variables with fixed variance  $P_N$  (corresponding to the noise power), constant for all CUEs.

Under the observation model (1), an energy detection (ED) scheme is the optimal detector in the Neyman–Pearson sense [20]. In particular, the following decision variable has to be evaluated:

$$W_k = \sum_{\ell=1}^m |\mu_k(\ell)|^2$$

and the binary decision of the CUE is given by

$$x_k = \begin{cases} 0 & \text{if } W_k < \tau \\ 1 & \text{if } W_k \geq \tau \end{cases} \quad (2)$$

where  $\tau$  is a properly selected decision threshold. In other words, each CUE decides for 0 if the channel is sensed idle, whereas it decides for 1 if the channel is sensed busy. The extension to soft decision rules goes beyond the scope of this paper and is the subject of our current research activity. The local FA and MD probabilities, under the proposed ED scheme, can be defined as follows:

$$P_{FA}^{(k)} \triangleq P(x_k = 1 | S_0) = P(W_k \geq \tau | S_0)$$

$$P_{MD}^{(k)} \triangleq P(x_k = 0 | S_1) = P(W_k < \tau | S_1).$$

Consequently, the correct detection (CD) probability is

$$P_{CD}^{(k)} = 1 - P_{MD}^{(k)}.$$

The  $k$ th CUE then generates its decision  $x_k \in \{0, 1\}$  on the absence ( $x_k = 0$ ) or presence ( $x_k = 1$ ) of a primary signal in the considered subchannel. This data is modulated using an antipodal modulation, e.g., binary phase shift keying, and transmitted to the secondary FC through a set of independent binary communication channels with additive white Gaussian noise. The received observable, at the secondary FC, from the  $k$ th CUE and associated with the status of the subchannel is

$$y_k = r_k + w_k \quad (3)$$

where  $r_k = 1 - 2x_k$  is the antipodal transmitted version of the local decision and  $w_k$  are i.i.d. Gaussian random variables with zero mean and variance  $\eta^2$ . Assuming unitary bit energy, the (bit) channel signal-to-noise ratio (SNR), assumed to be the same for all the CUE-FC communication channels,<sup>2</sup> can be expressed as

$$\text{SNR} = \frac{1}{\eta^2}.$$

At the FC, the observation vector on the subchannel status from the  $N$  CUEs is denoted as  $\mathbf{y} = (y_1, \dots, y_N)$ .

At this point, upon receiving the decisions from all the CUEs, the FC applies a proper fusion strategy to take a final decision on the status (idle or busy) of the subchannel. The FC can thus broadcast this information to all CUEs, possibly with the assignment of the idle subchannel to the largest possible number of CUEs, in order to minimize multiple access interference. The design of proper fusion rules, either in the absence or in the presence of information about nodes' positions, will be the subject of Section 5.

For the sake of clarity, all the acronyms used in the rest of the paper are summarized in Table 1.

### 3. Local sensing performance at a CUE

We first recall the performance from a single CUE perspective (see, for example, [15] and references therein). This is expedient to derive explicit expressions for the

<sup>2</sup> This is realistic in cellular systems, owing to power control.

**Table 1**

All the acronyms used in the paper.

PWN	Primary Wireless Network
CWN	Cognitive Wireless Network
PUE	Primary User Equipment
CUE	Cognitive User Equipment
AP	Access Point
FC	Fusion Center
FA	False Alarm
MD	Missed Detection
CD	Correct Detection
ROC	Receiver Operating Characteristic
ROI	Region Of Interest
ED	Energy Detection
PDF	Probability Density Function
i.i.d.	Independent and Identically Distributed
SNR	Signal-to-Noise Ratio

FA and MD probabilities, which will be used later to assess the gain brought by the knowledge of the nodes' positions. Being the subchannels independent, the sensing capabilities of each CUE and the corresponding FA and MD probabilities with ED do not depend on the considered subchannel (i.e., frequency band). Since each subchannel is assigned to at most one PUE, without loss of generality we also focus on a single PUE. Extending our formulation to a multi-subchannel scenario (e.g., bringing the medium access control protocol into the picture) is a very interesting research direction, but goes beyond the scope of the current paper.

Considering the complex plane as the bidimensional space of reference, we denote as  $v_k = v_k e^{j2\pi\theta_k}$  and  $v_p = v_p e^{j2\pi\phi}$  the positions of the  $k$ th CUE and the PUE, respectively, where  $0 \leq v_k, v_p \leq R$  and  $0 \leq \theta_k, \phi \leq 2\pi$ . The Euclidean distance  $d_k$  between the PUE and the  $k$ th CUE is

$$d_k = |v_p - v_k| = \sqrt{v_k^2 + v_p^2 - 2v_k v_p \cos(\theta_k - \phi)}.$$

Assuming a fixed transmit power  $P_T$  for primary nodes, the power  $P_R^{(k)}$  received by the  $k$ th CUE can be expressed, according to the Friis formula [17], as

$$P_R^{(k)} = \frac{K}{d_k^\alpha} h_k P_T$$

where:  $K$  is the gain at 1 m from the transmitter (i.e., the PUE);  $h_k$  is the log-normal shadowing coefficient of the link between the PUE and the  $k$ th CUE; and  $\alpha$  is the path-loss exponent ( $\alpha = 2 \div 4$  as in typical urban cellular scenarios [17]). Therefore, the sensing SNR experienced by the  $k$ th CUE, with respect to the PUE, can be expressed as follows:

$$\gamma_k(d_k, h) = \frac{P_R^{(k)}}{P_N} = \frac{K h_k P_T}{P_N d_k^\alpha}.$$

The local FA and MD probabilities at a CUE with ED can then be evaluated as [20]

$$P_{FA}^{(k)} = \Gamma_u(m\tau_N, m)$$

$$P_{MD}^{(k)} = 1 - \Gamma_u\left(\frac{m\tau_N}{1 + \gamma_k(d_k, h)}, m\right)$$

where  $\tau_N = P_N \tau$  is the normalized threshold with respect to the noise power (with  $\tau$  introduced in (2)) and

$\Gamma_u(a, n) \triangleq \int_a^\infty x^{n-1} e^{-x} dx / (n-1)!$  is the upper incomplete gamma function.

Note that the FA probability is the same for all CUEs and does not depend on their distances from the PUE: we thus denote  $P_{FA}^{(k)} = P_{FA}, \forall k \in \{1, 2, \dots, N\}$ . The MD probability, instead, depends on the distance  $d_k$  and on the shadowing term  $h_k$ . Averaging with respect to the statistical distribution of the shadowing term, the following expression for the average MD probability at distance  $d_k$  can be obtained:

$$\bar{P}_{MD}^{(k)}(d_k) = \mathbb{E}_{h_k} [P_{MD}^{(k)}(d_k, h_k)]$$

$$= 1 - \frac{1}{\sqrt{2\pi\sigma^2}}$$

$$\times \int_{-\infty}^{\infty} \Gamma_u\left(\frac{m\tau_N}{1 + \gamma_k(d_k, 10^{S/10})}, m\right) e^{-\frac{S^2}{2\sigma^2}} dS. \quad (4)$$

Note that, since the average MD probability in (4) is a function of  $d_k$  only, for notational simplicity we remove the superscript  $k$  and denote it as  $\bar{P}_{MD}(d_k)$  in the following.

#### 4. Information-theoretic framework

In this section, we derive an information-theoretic framework to predict the performance of the considered cognitive radio system and evaluate the benefits brought by the knowledge of the positions of the nodes in the network. In particular, we evaluate the average mutual information between the CUEs' observation vector at the FC, namely  $\mathbf{Y}$ , and the binary data representing the occupation status of a subchannel  $S \in \{0, 1\}$ .<sup>3</sup> Note that, owing to the independence between subchannels, we focus on a generic subchannel and, therefore, drop the subscript  $i$  for notational simplicity. To illustrate the benefits brought by the knowledge of nodes' positions at the FC, we consider four possible cases: (i) complete knowledge of the positions of *all* nodes' (CUEs and PUE); (ii) no knowledge of nodes' positions is available (blind case); (iii) only the positions of the CUEs are known; and (iv) only the position of the PUE is known. Note that knowledge of the PUE's position at the secondary FC (cases (i) and (iv)) is not of practical interest (otherwise, CWN and PWN would not be distinct), but may provide additional insights. Moreover, it is interesting from a theoretical perspective, since it allows to obtain meaningful performance benchmarks. From a practical point of view, however, it could be possible to estimate the position of the PUE through cooperation of the CUEs, e.g., by comparing the received signal strengths and exploiting the knowledge of the positions of the CUEs. A possible instance of this approach is proposed in [21].

##### 4.1. Complete knowledge of nodes' positions

In this scenario, we assume that the FC is aware of the position of the PUE and of all the CUEs in the ROI. In particular, let us denote by  $\mathbf{V} = (V_p, V_1, V_2, \dots, V_N)$  the

<sup>3</sup> In this section, uppercase letters are used to denote random variables, whereas lowercase letters are used for their realizations.

random vector, of size  $N + 1$ , representing all possible nodes' positions in the ROI and by  $\mathbf{v}$  a possible realization of it. The mutual information between  $S$  and  $\mathbf{Y}$ , conditioned on a particular realization  $\mathbf{v}$ , can be evaluated as

$$I_A(S; \mathbf{Y}|\mathbf{v}) = \left\{ \int_{\mathbf{y}} p(\mathbf{y}|S_0) \log_2 \left[ \frac{p(\mathbf{y}|S_0)}{p(\mathbf{y}|\mathbf{v})} \right] d\mathbf{y} \right\} P(S_0) + \left\{ \int_{\mathbf{y}} p(\mathbf{y}|S_1, \mathbf{v}) \log_2 \left[ \frac{p(\mathbf{y}|S_1, \mathbf{v})}{p(\mathbf{y}|\mathbf{v})} \right] d\mathbf{y} \right\} P(S_1). \quad (5)$$

Note that the probability density function (PDF) of  $\mathbf{Y}$ , conditioned on  $S_0$ , does not depend on  $\mathbf{v}$ . In order to approximately compute (5),<sup>4</sup> one may resort to Monte Carlo simulations generating, at each run, a realization of  $\mathbf{v}$ . We also generate a realization of vector  $\mathbf{y}$  considering only the status  $S_0$ : the obtained vector is denoted as  $\mathbf{y}^{(0)}$ . Likewise, we generate a realization of vector  $\mathbf{y}$  considering only the status  $S_1$ : the obtained vector is denoted as  $\mathbf{y}^{(1)}$ . Therefore, (5) can be approximated as follows:

$$I_A(S; \mathbf{Y}|\mathbf{v}) \simeq \mathbb{E}_{\mathbf{Y}^{(0)}} \left\{ \log_2 \left[ \frac{p(\mathbf{y}^{(0)}|S_0)}{p(\mathbf{y}|\mathbf{v})} \right] \right\} P(S_0) + \mathbb{E}_{\mathbf{Y}^{(1)}} \left\{ \log_2 \left[ \frac{p(\mathbf{y}^{(1)}|S_1, \mathbf{v})}{p(\mathbf{y}|\mathbf{v})} \right] \right\} \quad (6)$$

where  $\mathbb{E}_{\mathbf{Y}_i}[\cdot]$  denotes the average with respect to the random vector  $\mathbf{Y}$  conditioned on status  $i \in \{0, 1\}$ . The subscript "A" in the mutual information refers to the fact that the positions of "all" nodes' (CUEs and PUE) are assumed to be known.

Under the assumption of independent noise and shadowing at each CUE, the conditional PDFs in (6) can be evaluated as

$$p(\mathbf{y}^{(0)}|S_0) = \prod_{k=1}^N p(y_k^{(0)}|S_0) = \prod_{k=1}^N \frac{1}{\sqrt{2\pi\sigma^2}} \left\{ (1 - P_{FA}) e^{-\frac{(y_k^{(0)}-1)^2}{2\sigma^2}} + P_{FA} e^{-\frac{(y_k^{(0)}+1)^2}{2\sigma^2}} \right\} \quad (7)$$

$$p(\mathbf{y}^{(1)}|S_1, \mathbf{v}) = \prod_{k=1}^N p(y_k^{(1)}|S_1, \mathbf{v}) = \prod_{k=1}^N \frac{1}{\sqrt{2\pi\sigma^2}} \left\{ \bar{P}_{MD}(d_k) e^{-\frac{(y_k^{(0)}-1)^2}{2\sigma^2}} + [1 - \bar{P}_{MD}(d_k)] e^{-\frac{(y_k^{(1)}+1)^2}{2\sigma^2}} \right\} \quad (8)$$

where we have used the fact that the conditional PDFs  $p(y_k^{(0)}|S_0)$  and  $p(y_k^{(1)}|S_1, \mathbf{v})$  can be written, according to (1)

<sup>4</sup> With infinitely long simulations, the Monte Carlo-based computation of (5) would be exact.

and (2), as weighted sums of Gaussian random variables. Applying the total probability theorem, from (7) and (8) it follows that

$$p(\mathbf{y}|\mathbf{v}) = p(\mathbf{y}^{(0)}|S_0) P(S_0) + p(\mathbf{y}^{(1)}|S_1, \mathbf{v}) P(S_1).$$

#### 4.2. No knowledge of nodes' positions

In this scenario, the secondary FC has no knowledge about the actual nodes' (both CUEs' and PUE's) deployment inside the ROI. Following the approach proposed in Section 4.1, the mutual information can now be expressed as

$$I_B(S; \mathbf{Y}) = \left\{ \int_{\mathbf{y}} p(\mathbf{y}|S_0) \log_2 \left[ \frac{p(\mathbf{y}|S_0)}{p(\mathbf{y})} \right] d\mathbf{y} \right\} P(S_0) + \left\{ \int_{\mathbf{y}} p(\mathbf{y}|S_1) \log_2 \left[ \frac{p(\mathbf{y}|S_1)}{p(\mathbf{y})} \right] d\mathbf{y} \right\} P(S_1) \quad (9)$$

where subscript "B" stands for "blind" scenario. The conditional PDF  $p(\mathbf{y}|S_0)$  in (9) can be approximated, by Monte Carlo simulations, as in (7). The other conditional PDF  $p(\mathbf{y}|S_1)$ , instead, cannot be directly approximate and, therefore, will be approximated by a "manageable" PDF, indicated as  $q(\mathbf{y}|S_1)$ , which underlies a different channel model. Resorting to the theory of mismatched communications, the mutual information can be approximated as [22]

$$I_B(S; \mathbf{Y}) \simeq \left\{ \int_{\mathbf{y}} p(\mathbf{y}|S_0) \log_2 \left[ \frac{p(\mathbf{y}|S_0)}{q(\mathbf{y})} \right] d\mathbf{y} \right\} P(S_0) + \left\{ \int_{\mathbf{y}} q(\mathbf{y}|S_1) \log_2 \left[ \frac{q(\mathbf{y}|S_1)}{q(\mathbf{y})} \right] d\mathbf{y} \right\} P(S_1). \quad (10)$$

As in Section 4.1, resorting to a Monte Carlo simulation to numerically approximate (10) leads to

$$I_B(S; \mathbf{Y}) \simeq \mathbb{E}_{\mathbf{Y}^{(0)}} \left\{ \log_2 \left[ \frac{p(\mathbf{y}^{(0)}|S_0)}{q(\mathbf{y})} \right] \right\} P(S_0) + \mathbb{E}_{\mathbf{Y}^{(1)}} \left\{ \log_2 \left[ \frac{q(\mathbf{y}^{(1)}|S_1)}{q(\mathbf{y})} \right] \right\} P(S_1) \quad (11)$$

where

$$q(\mathbf{y}) = p(\mathbf{y}^{(0)}|S_0) P(S_0) + q(\mathbf{y}^{(1)}|S_1) P(S_1).$$

We remark that, at this point, the only unknown term in (11) is  $q(\mathbf{y}^{(1)}|S_1)$ , which can be computed by averaging over the distribution of the PUE's position. We now propose a simple derivation of this distribution.

We derive the PDF of the distance between the PUE and the CUE for a given PUE's position. Owing to the symmetry of the problem, we can assume, without loss of generality, that the PUE lies on the real axis, as shown in Fig. 2. Let us assume, for notational simplicity, that  $R = 1$ . It follows that, for  $\rho < R - \xi = 1 - \xi$  (case 1 on the right side of Fig. 2), the annulus centered at  $\xi$  with inner radius  $\rho - \delta\rho/2$  and outer radius  $\rho + \delta\rho/2$  is fully included into the unit circle and, therefore, the PDF of  $\rho$  can be expressed as

$$f_d(\rho) = \begin{cases} 2\rho & 0 \leq \rho < 1 - \xi \\ 0 & \text{otherwise.} \end{cases}$$



Note that, using similar considerations, it can be shown that the PDF of the PUE's position (assuming that it lies as well on the  $x$  axis) can be written as follows:

$$f_v(\xi) = \begin{cases} 2\xi & \xi \geq 0 \\ 0 & \text{otherwise.} \end{cases} \quad (12)$$

On the other hand, for  $\rho \geq 1 - \xi$ , only a portion of the annulus lies within the unit circle and the PDF is given by the ratio between such portion and the surface of the unit circle (case 2 on the left side of Fig. 2). Using simple geometric considerations, it is then straightforward to obtain

$$f_d(\rho, \xi) = \begin{cases} 2\rho & 0 \leq \rho \leq 1 - \xi \\ 2\rho \left[ 1 - \frac{1}{\pi} \arccos \left( \frac{1 - \xi^2 - \rho^2}{2\xi\rho} \right) \right] & 1 - \xi < \rho \leq 1 + \xi \\ 0 & \text{otherwise.} \end{cases} \quad (13)$$

At this point, one can write

$$q(\mathbf{y}^{(1)}|S_1) = \int_{\xi=0}^1 q(\mathbf{y}^{(1)}|S_1, \xi) f_v(\xi) d\xi \quad (14)$$

where  $f_v(\xi)$  is given by (12). In order to compute  $q(\mathbf{y}^{(1)}|S_1, \xi)$ , we should observe that, since the positions of the PUE and of the CUEs are i.i.d. within the ROI, for a given PUE's position, the local decisions  $\{x_k\}_{k=1}^N$  are conditionally i.i.d. and, therefore,

$$q(\mathbf{y}^{(1)}|S_1, \xi) = \prod_{k=1}^N \frac{1}{\sqrt{2\pi\sigma^2}} \left\{ \check{P}_{\text{MD}}(\xi) e^{-\frac{(y_k^{(0)} - 1)^2}{2\sigma^2}} + \left[ 1 - \check{P}_{\text{MD}}(\xi) \right] e^{-\frac{(y_k^{(1)} + 1)^2}{2\sigma^2}} \right\} \quad (15)$$

where  $\check{P}_{\text{MD}}(\xi)$  is the average MD probability of each CUE for a given position  $\xi$  of the PUE, which can be evaluated by averaging over the PDF of the distance, given by (13), between the CUE and the PUE, i.e.,

$$\check{P}_{\text{MD}}(\xi) = \int_{\rho} \bar{P}_{\text{MD}}(\rho R) f_d(\rho, \xi) d\rho \quad (16)$$

where  $\bar{P}_{\text{MD}}(\rho R)$  can be computed according to (4).

#### 4.3. Knowledge of CUEs' positions

Denoting as  $\mathbf{V}_c = (V_1, V_2, \dots, V_N)$  the random vector of CUEs' positions, one can estimate  $p(\mathbf{y}|S_1, \mathbf{v}_c)$  in (8), by still resorting to the theory of mismatched decoding, as

$$\check{q}(\mathbf{y}|S_1, \mathbf{v}_c) = \int_{\text{ROI}} p(\mathbf{y}|S_1, \mathbf{v}_c, v_p) f(v_p) dv_p.$$

As already done for the "blind" scenario in Section 4.2, the mutual information becomes

$$I_c(S; \mathbf{Y}|\mathbf{v}_c) \simeq \mathbb{E}_{\mathbf{Y}^{(0)}} \left\{ \log_2 \left[ \frac{p(\mathbf{y}^{(0)}|S_0)}{\check{q}(\mathbf{y}|\mathbf{v}_c)} \right] \right\} P(S_0) + \mathbb{E}_{\mathbf{Y}^{(1)}} \left\{ \log_2 \left[ \frac{\check{q}(\mathbf{y}^{(1)}|S_1, \mathbf{v}_c)}{\check{q}(\mathbf{y}|\mathbf{v}_c)} \right] \right\} P(S_1)$$

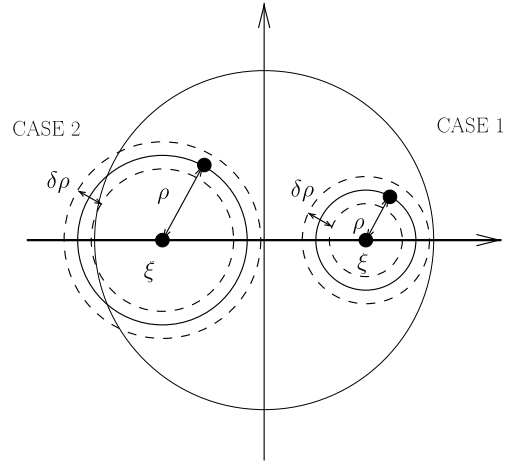


Fig. 2. Illustrative example for the computation of the distance PDF.

where subscript "C" denotes the fact that only CUEs' positions are available and

$$\check{q}(\mathbf{y}|\mathbf{v}_c) = p(\mathbf{y}^{(0)}|S_0) P(S_0) + \check{q}(\mathbf{y}^{(1)}|S_1, \mathbf{v}_c) P(S_1).$$

#### 4.4. Knowledge of the PUE's position

Using similar mathematical passages as in the previous cases and using the subscript "P" to denote the fact that only the PUE's position is available, one has

$$I_P(S; \mathbf{Y}|v_p) \simeq \mathbb{E}_{\mathbf{Y}^{(0)}} \left\{ \log_2 \left[ \frac{p(\mathbf{y}^{(0)}|S_0)}{\check{q}(\mathbf{y}|v_p)} \right] \right\} P(S_0) + \mathbb{E}_{\mathbf{Y}^{(1)}} \left\{ \log_2 \left[ \frac{\check{q}(\mathbf{y}^{(1)}|S_1, v_p)}{\check{q}(\mathbf{y}|v_p)} \right] \right\} P(S_1)$$

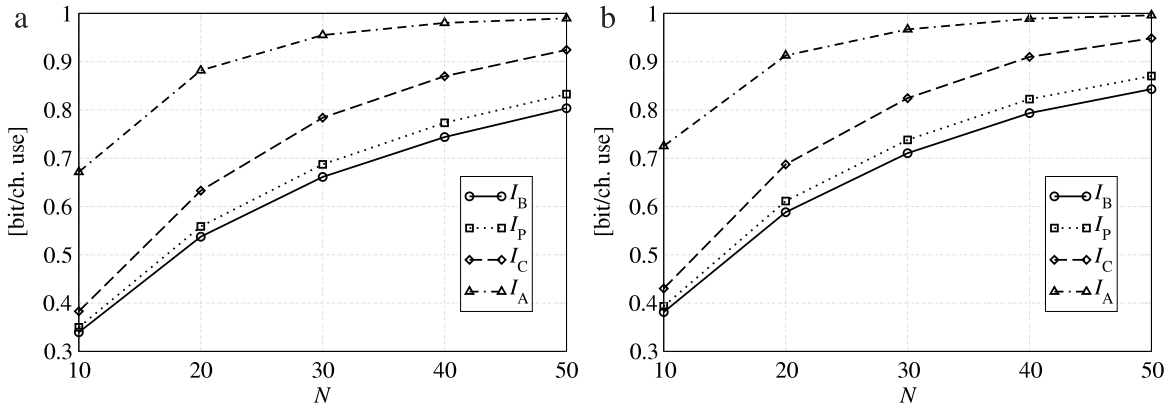
where  $\check{q}(\mathbf{y}^{(1)}|S_1, v_p)$  has the same meaning of the PDF  $q(\mathbf{y}^{(1)}|S_1, \xi)$  already derived in (15) and

$$\check{q}(\mathbf{y}|v_p) = p(\mathbf{y}^{(0)}|S_0) P(S_0) + \check{q}(\mathbf{y}^{(1)}|S_1, v_p) P(S_1).$$

#### 4.5. Numerical results

We now present results for the mutual information to show the potential benefits brought by the knowledge of information about nodes' positions in the network. To this end, we consider an illustrative scenario of realistic wireless cellular networking, characterized by the following main system parameters:  $R = 1$  km,  $P_T = 30$  mW,  $\alpha = 4$ ,  $\sigma = 5$  dB,  $P_N = -110$  dBm, and  $m = 10$ . In order to obtain statistically accurate performance results, Monte Carlo simulations are averaged over 5000 different trials, with independent realizations of the shadowing terms. Note that similar considerations can be carried out for different values of such parameters.

It is worth noting that, for all the considered scenarios, the system performance depends on the values of the normalized local threshold  $\tau_N$ . Note that a small value of  $\tau_N$  yields a high FA probability, whereas a large



**Fig. 3.** Mutual information, as a function of the number  $N$  of CUEs, for all scenarios described in Sections 4.1–4.4. Different communication scenarios are considered: (a) noisy communications with SNR = 5 dB and (b) error-free communications.

value of  $\tau_N$  entails a high MD probability. The optimized value of  $\tau_N$  obviously depends on the sensing SNR experienced by each CUE, which, in turn, depends on several other uncontrollable factors, such as the positions of CUEs/PUEs and shadowing. We preliminarily observe that the selection of a different and optimized threshold for each CUE would involve a huge amount of message exchange between the FC and the CUEs. Therefore, we make the reasonable assumption that the threshold is a predefined system parameter to be optimized off-line (e.g., in a training phase) assuming no knowledge about nodes' positions. Henceforth, we set  $\tau_N = 1.5$  so that, in the considered settings, the average per-node CD probability is, according to (4), approximately equal to 0.3. We remark, however, that the proposed approach is general: should one have more stringent constraints (e.g., lower CD probability), the threshold  $\tau_N$  should correspondingly be optimized.

In Fig. 3, the mutual information is shown, as a function of the number  $N$  of CUEs, for all scenarios described in Sections 4.1–4.4. Two possible values for the communication SNR are considered: (a) 5 dB and (b)  $+\infty$ . The latter ideal case corresponds to error-free communications from the CUEs to the FC. It can be observed that, in both cases, knowledge of all nodes' positions allows to obtain, as expected, the best performance. Moreover, the improvement brought by the knowledge of the PUE's position, with respect to the case with no knowledge, is minor. On the other hand, knowledge of the CUEs' positions leads to a major performance improvement with respect to the blind case and the mutual information is close, especially for large values of  $N$ , to that with knowledge of all nodes' (CUEs' and PUE's) positions. By comparing Fig. 3(a) with Fig. 3(b), it can be observed that increasing the communication SNR has a limited impact: this indicates that the performance is mostly influenced by the available information to be fused at the FC. Finally, as expected, increasing the number  $N$  of CUEs or the SNR improves the performance, thanks to the larger amount of information, with better quality, available at the FC.

## 5. Classical ROC approach

While the focus of Section 4 was on information-theoretic limits, we now set to investigate the performance

of practical systems, i.e., we consider a “classical” ROC approach. In order to do this, we first derive optimal fusion rules at the FC. In particular, we only consider two cases regarding the availability of information about nodes' positions: (i) blind scenario (no position information at the FC) and (ii) knowledge of the CUEs' positions. We do not consider the case with knowledge of the PUE's position, since this entails a minor performance improvement, as observed in Fig. 3. Moreover, it is not realistic that the FC of the secondary network is aware of the position of the PUE, which belongs to a different network.

We consider the case where data, i.e., local decisions  $\{x_k\}_{k=1}^N$  on the presence/absence of the PUE's signal in the considered subchannel at the CUEs, are transmitted, without channel coding, by the CUEs to the secondary FC, using a set of orthogonal error-free communication channels.<sup>5</sup> In fact, the results in Fig. 3 show that, for the considered value of  $\tau_N$ , the performance slightly depends on the communication SNR.

The optimal fusion strategy at the FC stems from the application of the Neyman–Pearson criterion and requires the evaluation of the likelihood ratio (LR) between the probabilities of the decisions  $\{x_k\}_{k=1}^N$  received from the nodes (recall that error-free communications are considered, i.e.,  $y_k = x_k$ ,  $k = 1, \dots, N$  in (3)) given either of the two hypotheses  $S_1$  and  $S_0$ , i.e.,

$$\Lambda = \frac{P(x_1, \dots, x_N | S_1)}{P(x_1, \dots, x_N | S_0)} \stackrel{S_1}{\geq} \lambda \quad (17)$$

where  $\lambda$  is a proper fusion threshold to be optimized. We now specialize (17) for the two cases of interest. In particular, in Section 5.1 we first focus on a scenario where the FC has no knowledge of the positions of the CUEs. Then, in Section 5.2 we assume that the positions of the CUEs are available at the FC.

<sup>5</sup> In other words, we assume the use of a medium access control which avoids collisions between CUEs' transmissions, e.g., time division multiple access.

### 5.1. Unknown CUEs' positions

We first consider the case where the FC is not aware of any information about the nodes' positions in the ROI. Assuming blind detection, i.e., the FC has no knowledge about the effective sensing accuracy (in terms of local FA and MD probabilities) of each CUE, and since the communication channels between the CUEs and the secondary FC are error-free, the transmitted data are received correctly and the fusion rule (17) reduces to the following [23]:

$$\Lambda = \sum_{k=1}^N x_k \underset{S_0}{\overset{S_1}{\geq}} T \quad (18)$$

where  $T$  is a "global" decision threshold (i.e., equal for all subchannels) to be optimized. In other words, the hypothesis testing problem turns out to be a counting problem: for example, the number of local decisions, at the CUEs, in favor of  $S_1$  is first counted and then compared with the threshold  $T$ .

### 5.2. Known CUEs' positions

We now consider the case where the positions of the CUEs in the ROI, denoted as  $\{v_k\}_{k=1}^N$ , are perfectly known to the FC. This may be reasonable if the CUEs transmit, in their packets, information about their positions (e.g., based on acquired GPS data) or if the positions can be inferred using localization techniques (e.g., triangulation based on the received signal powers from anchor nodes, such as cellular base stations).

Assume that the position of the PUE is known and denote it as  $v_p$ . Under this assumption, one could easily evaluate the conditional probabilities of the decisions  $\{x_k\}_{k=1}^N$  at the CUEs. Indeed, according to the system model described in Section 2, for a given position  $v_p$  of the PUE, the decisions  $\{x_k\}_{k=1}^N$  are independent and it holds that

$$P(x_k = 1 | S_1, v_p) = \bar{P}_{MD}^{(k)}(d_k)$$

where  $d_k = |v_k - v_p|$ , and

$$P(x_k = 1 | S_0) = P_{FA}$$

Therefore, one can compute the numerator of (17) conditioned on the position  $v_p$  as

$$\begin{aligned} P(x_1, \dots, x_N | S_1, v_p) \\ = \prod_{k \in \mathcal{X}_1} [1 - \bar{P}_{MD}^{(k)}(d_k)] \prod_{q \in \mathcal{X}_0} \bar{P}_{MD}^{(q)}(d_q) \end{aligned} \quad (19)$$

where

$$\mathcal{X}_0 \triangleq \{h \in \{1, \dots, N\} : x_h = 0\}$$

$$\mathcal{X}_1 \triangleq \{j \in \{1, \dots, N\} : x_j = 1\}.$$

Averaging (19) with respect to the position of the PUE, one can express the unconditional probability at the numerator of (17) as

$$\begin{aligned} P(x_1, \dots, x_N | S_1) \\ = \int_{\text{ROI}} P(x_1, \dots, x_N | S_1, v_p) f(v_p) dv_p \end{aligned} \quad (20)$$

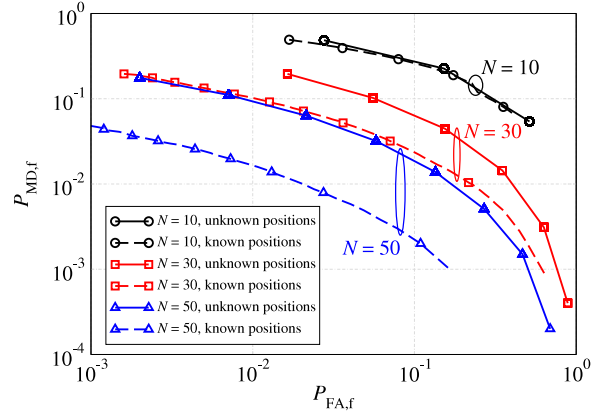


Fig. 4. ROC curves for various values of  $N$  (namely 10, 30, and 50), considering known and unknown positions of the CUEs.

where  $f(v_p)$  is the PDF of the PUE position in the ROI, i.e., uniform in the circular cell. The integral (20) does not have a closed-form expression, but can be numerically solved. Therefore, in this case closed-form expressions for  $P_{CD,f}$  and  $P_{FA,f}$  cannot be obtained and one needs to resort to simulations to evaluate the system performance. However, proper sub-optimal fusion strategies could be devised to avoid the computation of the integral in (20): this goes beyond the scope of this paper and is the subject of our current research activity. The denominator of (17) can readily be expressed as follows:

$$P(x_1, \dots, x_N | S_0) = P_{FA}^{N_1} (1 - P_{FA})^{N-N_1}$$

where  $N_1 \triangleq |\mathcal{X}_1|$ .

### 5.3. Numerical results

We now present numerical results for the practical scenario of interest in this section. In all simulated scenarios (with and without knowledge of the CUEs' positions), the performance is investigated in terms of ROC curves, for the binary hypothesis testing problem of interest, showing (in a log-log scale) the final (at the secondary FC) MD probability as a function of the final FA probability [12]. In the ROC curve, each point corresponds to a different value of the global fusion thresholds, namely:  $T$  in the case with unknown CUEs' positions (Section 5.1) and  $\lambda$  in the case with known CUEs' positions (Section 5.2).

In Fig. 4, ROC curves are shown for various values of  $N$  (namely 10, 30, and 50), considering both the cases with known and unknown positions of the CUEs. It can be observed that the knowledge of the positions of the CUEs allows, through the use of the optimal fusion rule outlined in Section 5.2, to improve the performance for all values of  $N$ . This is to be expected, since the secondary FC can rely on a larger amount of (statistical) information to take the final decision. In particular, the larger the number of CUEs, the more relevant the performance improvement. Similar results, not shown here for conciseness, hold for other communication scenarios with different wireless propagation characteristics.



## 6. Concluding remarks

In this paper, we have analyzed cognitive wireless networking, where CUEs sense the frequency spectrum (divided into subchannels freely assigned to PUEs) and cooperate to estimate the idle (primary) subchannels which can be used to transmit their data. First, a simple analytical framework for characterizing the local sensing performance, in terms of MD and FA probabilities per subchannel at a CUE, has been derived. Then, we have computed the mutual information between the occupation status and the observations at the FC, with and without knowledge of the positions of the nodes in the network, showing a potential significant benefit brought by the use of this information. Finally, we have derived optimal fusion rules with and without knowledge of CUEs' positions in the ROI, in the presence of uncoded transmissions over error-free communication channels between CUEs and the secondary FC. Our results show a performance improvement brought by the knowledge of the CUEs' positions. This performance improvement is more pronounced the larger is the number of CUEs, as cooperation in channel sensing becomes more effective.

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