Modeling Nonlinearity in Coherent Transmissions with Dominant Interpulse-Four-Wave-Mixing

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Abstract: We provide a new analytical model to predict the nonlinear interference coefficient and the nonlinear threshold in coherent transmissions with dominant single-channel IFWM.

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1 Introduction

It has recently been shown that, in high bit-rate coherent optical links with no dispersion management (NDM), the nonlinear interference (NLI) is a zero-mean additive circular complex-Gaussian noise, independent of the symbol of interest, already after a few spans [1]. Based on such a powerful observation, a nonlinear Gaussian model for NDM coherent communications was proposed [2–4]. In this paper, we wish to extend those studies to the regime in which single-channel inter-pulse four wave mixing (IFWM) is the dominant nonlinearity. This regime includes both dispersion-managed (DM) and NDM links at sufficiently large baud-rates.

2 Nonlinear Gaussian Model

Consider a single-channel long-haul optical link with dual polarization coherent reception. Assume that both the amplified spontaneous emission (ASE) and the NLI are independent additive complex-Gaussian noises. After coherent reception with polarization demultiplexing and ideal linear electrical equalization, followed by matched filtering with ideal carrier estimation, the 2-dimensional (2D) sampled received complex field vector is: $\underline{r}(t) = \sqrt{P}\underline{U}(t) + \underline{n}_L(t) + \underline{n}_{NL}(t)$, where P [W] is the signal average power, $\underline{U}$ the normalized signal vector, $\underline{n}_L$ the ASE, and $\underline{n}_{NL}$ the NLI. The electrical signal-noise ratio (SNR) at the decision gate is

$$S = \frac{P}{N_A + N_{NL}} \tag{1}$$

where $N_A = Var[\underline{n}_L] = \beta N$ is the ASE power, which linearly increases with the number of spans N, and $N_{NL} = Var[\underline{n}_{NL}] = a_{NL}P^3$ is the NLI power, obtained from a first-order regular perturbation [2,4]. The main goal of this paper is to provide a general analytical expression of the NLI coefficient a_{NL}, valid for dominant IFWM. Such an expression will be used to analytically cross-validate recent simulation results on nonlinear threshold (NLT) [5].

3 Nonlinear Threshold

We define the *constrained* NLT at reference BER_0 (i.e., at its corresponding format-dependent SNR S_0) as the transmitted power P_{NLT} yielding the maximum of the "bell-curve" S versus P, where the maximum value is *constrained* to S_0. Maximization of (1) with ASE noise adjusted such that the top value is $S = S_0$ yields [2]

$$P_{NLT} = \frac{1}{(3S_0 a_{NL})^{1/2}} \tag{2}$$

and depends only on S_0 and a_{NL}. It has been shown that the model (1), at the top S value, yields an SNR penalty with respect to linear propagation of 1.76 dB [2,3]. We can prove that the 1dB NLT P_1, i.e., the transmitted power needed to achieve S_0 with 1 dB of SNR penalty, is 1.05 dB smaller than P_{NLT}. P_1 corresponds to the NLT simulated in [5] that we wish to double-check with our theory.

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4 Nonlinear Interference coefficient

We now describe a procedure to derive closed-form analytical expressions of the NLI coefficient a_{NL}. The NLI on each polarization tributary ($i = x$ or y) can be obtained from a first-order regular perturbation as [6,7]:

$$n_{NL,i} = j\sqrt{P}\Phi_{NL} \iint_{-\infty}^{\infty} \eta(t_1t_2)U_i(t+t_1)U_i(t+t_1+t_2)U_i(t+t_2)dt_1dt_2 \quad (3)$$

where: the nonlinear phase is $\Phi_{NL} \triangleq P\int_0^L \gamma(s)G(s)\mathrm{d}s$, with γ the fiber nonlinear coefficient and $G(s)$ the power gain at coordinate s; $\eta(t_1t_2)$ is the time-domain kernel (time is normalized to the symbol time $1/R$, where R is the baud-rate), whose 2D Fourier transform is

$$\tilde{\eta}(w) \triangleq \frac{\int_0^L \gamma(s)G(s)\mathrm{e}^{-jC(s)w}\mathrm{d}s}{\int_0^L \gamma(s)G(s)\mathrm{d}s}$$

where: $w = \omega_1\omega_2$; L is the total link length; and the normalized cumulated dispersion (NCD) is $C(s) = -R^2\int_0^s \beta_2(z)\mathrm{d}z$, where β_2 is the fiber chromatic dispersion, and zero dispersion slope is assumed. For a linear digital modulation we have $U_i(t) = \sum_{k=-\infty}^{\infty} s_k p(t-k)$ where s_k is the complex information symbol (on polarization i) transmitted in the k-th symbol interval, and $p(t)$ is the supporting pulse. As done in [6,7], when the time-domain kernel is much broader than the symbol time and thus quasi-constant over squares of size 1 in the normalized time plane (t_1,t_2), then the NLI term in (3), for a link with spans much longer than $1/\alpha$ and lumped amplification, simplifies to $n_{NL,i} = c_{NL}P^{3/2}$, with

$$c_{NL} = j\frac{\gamma}{\alpha}N\sum_{m,n,l} s_m s_n s_l^* \eta\left((m-l)(n-l)\right) \quad (4)$$

where the summation accounts for IFWM terms, i.e., is over all m,n,l such that $m+n = l$, with $m \neq l$, $n \neq l$. The NLI power in (1) comes from both polarizations and is $N_{NL} \triangleq \eta_p E[|n_{NL}|^2] = \eta_p E[|c_{NL}|^2]P^3$, where $\eta_p = 2$ for independent NLI from each polarization. Thus $a_{NL} = \eta_p E[|c_{NL}|^2]$, where the expectation is taken over the random symbols. For *any* modulation format with $E[s_k] = 0$ and $E[|s_k|^2] = 1$, we get

$$a_{NL} = \eta_p(\frac{\gamma}{\alpha}Nd_f)^2 2\sum_{p=1}^{\infty}\sum_{q=1}^{\infty}|\eta(pq)|^2 \quad (5)$$

where $p \triangleq n-l$, $q \triangleq m-l$, and $d_f = 2$ is the degeneracy factor. The time kernel magnitude decreases and eventually vanishes after an "effective" time duration τ_M. Since each $|\eta(pq)|^2$ in the double summation in (5) is actually an approximation of the double integral of the kernel over a square of edge 1 centered at the point (p,q), we can approximate the double summation as a double integral over the domain $\mathscr{D}$ of the (t_1,t_2) plane delimited by the hyperbola $t_1t_2 = \tau_M$, the vertical line passing through $t_1 = 1/2$, and the horizontal line passing through $t_2 = 1/2$. We can thus upper-bound the coefficient as

$$a_{NL} \leq \eta_p(\frac{\gamma}{\alpha}Nd_f)^2 2\ln(4\tau_M)\left[\int_0^{\infty}|\eta(\tau)|^2\mathrm{d}\tau\right] \quad (6)$$

and what we need is an expression of the kernel duration τ_M, and of the above integral of the kernel magnitude. We may choose $\tau_M \triangleq \mu\tau_{rms}$ for some positive multiplier μ of the r.m.s. width $\tau_{rms}^2 = \int_{-\infty}^{\infty}\tau^2|\eta(\tau)|^2\mathrm{d}\tau/\int_{-\infty}^{\infty}|\eta(u)|^2\mathrm{d}u$. We chose $\mu = 1.5$ in all numerical results. Now, an analytical expression of the time kernel is not known even for the simplest links, except for lossless links [7]. However, there is a nice trick. For every optical link, both with and without dispersion management, a physically meaningful function is the *power-weighted dispersion distribution* (PWDD) $J(c)$, representing signal power versus NCD c, which was shown to be the inverse 1D-Fourier transform: $J(c) = \mathscr{F}^{-1}[\tilde{\eta}(w)]$ [7]. One also has that: $\eta(\tau) = \mathscr{F}^{-1}\left[\frac{1}{|\omega|}J(\frac{1}{\omega})\right]$, where $\tau = t_1t_2$ [6,7]. Because of the Fourier relationship between $J(c)$ and $\eta(\tau)$, we can prove that $2\int_0^{\infty}|\eta(\tau)|^2d\tau = \int_{-\infty}^{\infty}J^2(c)dc$, and that $\tau_{rms}^2 = \int_{-\infty}^{\infty}[J(c)+cJ'(c)]^2\mathrm{d}c/\int_{-\infty}^{\infty}J^2(c)\mathrm{d}c$, where $J'(c) = \frac{d}{dc}J(c)$. Hence, a_{NL} in (6) can be expressed solely in terms of integrals of $J(c)$. Note also that it applies to *any* zero-mean modulation format. We managed to get closed-form expressions of the a_{NL} upper-bound (6) for several links of interest. For instance, for NDM links we got for $N \gtrsim 5$:

$$a_{NL} \leq \eta_P(\frac{\gamma}{\alpha})^2\frac{N}{\pi\mathscr{S}}\ln(\frac{4\mu}{\sqrt{5}}(\alpha\ell N)^2\mathscr{S})$$

where ℓ is span length, and $\mathscr{S} \triangleq \frac{|\beta_2|}{\alpha}R^2$ is fiber strength. Note the similarity of this expression with that of a Nyquist-WDM NDM system derived in [3] using a frequency-domain approach. The major difference is the $N\log N$ scaling law in the IFWM-dominated regime, as opposed to the simpler N scaling when presumably cross-nonlinearities dominate.

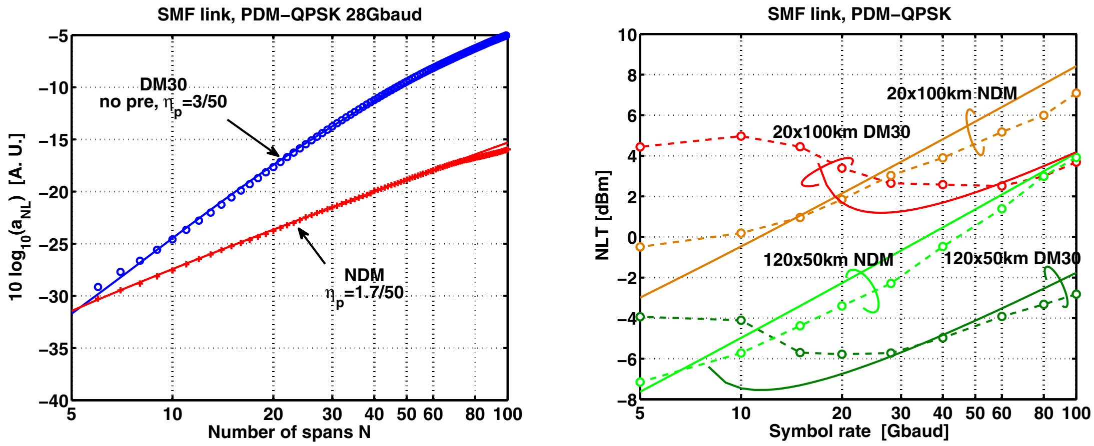

Figure 1. **(Left)** a_{NL} [dB] versus spans N from eq. (6) (solid) and simulations (symbols). PDM-QPSK on Nx100 km SMF links, R=28 Gbaud. **(Right)** 1dB NLT vs. symbol rate R for: i) theory $P_1 = P_{NLT} - 1.05$ dBm (solid, eq. (2)); ii) simulations from [5]. DM30 = DM with 30 ps/nm RDPS.

5 Results

Fig. 1(left) shows a plot of the a_{NL} formula (6) versus number of spans N (solid), and numerically simulated values (symbols), for a 28 Gbaud polarization-division multiplexed quadrature phase shift keying (PDM-QPSK) coherent format over single mode fiber (SMF, $\beta_2 = -21\, ps^2/nm$) for an Nx100 km link, both NDM and DM with 30 ps/nm/km (DM30) of residual dispersion per span (RDPS) and no pre-compensation. A fitting factor $\eta_p = 3/50$ was used for DM, and $\eta_p = 1.7/50$ for NDM. We appreciate the match of theory and simulation, as well as the announced $N \log N$ scaling law in the NDM case. The perceived NDM slope over a 50 span range is ~ 1.25 dB/dB as in [1], although restricting the range to the first 15 spans gives ~ 1.35 dB/dB, as we experimentally verified in a companion study. NLI grows faster in the DM case: a_{NL} has an initial slope of ~ 2 dB/dB and then bends at larger N.

Fig. 1(right) shows the 1dB NLT at $BER_0 = 10^{-3}$ versus baud-rate for a PDM-QPSK format for both NDM, and a DM30 link with optimized pre-compensation, both at 20x100 km and at 120x50 km distance. Symbols refer to single-channel simulation results taken from [5], solid lines to the formula $P_1 = P_{NLT} - 1.05$ dBm using (2) and the same η_p fitting factors as in Fig. 1(left). While for DM links theory only captures the general trend versus R with major discrepancies at lower R where IFWM is not dominant, the match in NDM links (optimized at 28 Gbaud through the fitting factors η_p) is more reasonable and improves as the number of spans N increases.

6 Conclusions

We have provided a new model of NLI in IFWM dominated links, which reasonably models NDM links, as well as high baud-rate DM links. Such a model provides a quick qualitative tool to compare transmission link parameters in terms of their impact on received SNR.

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